MRCET CAMPUS

ICET CODE MLRD

QUANTITATIVE ANALYSIS & BUSINESS DECISIONS

Digital Notes

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Advice to the Students:

1. Ensure you have a strong grasp of algebra, calculus, and basic mathematical principles.
2. Understand how to formulate and solve linear programming problems, including concepts like constraints, objective functions, and the feasible region.
3. Get familiar with decision trees, payoff tables, and the concept of expected value.
4. Study network optimization problems such as the shortest path, maximum flow, and minimum cost flow problems.
5. Solve past exam papers or sample questions to get familiar with the exam format and types of questions.

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution-UGC, Govt. of India)

DEPARTMENT OF BUSINESS MANAGEMENT

Course Name :MBA I Year II SEM
Name of the Subject: Quantitative Analysis and Business Decisions
Academic year : 2024-2025
Prescribed Textbook: J.K.Sharma,“Operations Research: Theory and applications.
Nature of the Subject: Common paper

Course Aim: Quantitative methods-research techniques used to analyze quantitative data-enable professionals to organize and understand numbers and, inturn, to make good decisions.

Learning Objective: *Quantitative Methods An Introduction for Business Management* presentstheapplicationofquantitativemathematicalmodelingtodecisionmakinginabusinessmanagement context and emphasizes the role of data in drawing conclusions, and also the pitfallsofundiscerningrelianceofsoftwarepackagesthatimplementstandardstatisticalprocedures.Withhands-onapplicationsandexplanationsandsuccessfullyoutlinesthenecessarytoolstomakesmart andsuccessfulbusinessdecisions.

Unit-1:Nature and Scope of Operations Research

OriginsofOR,ApplicationsofORindifferentManagerialareas,Definingamodel,typesofmodel,Process for developing an operations research model, Practices, opportunities and short comings of using an OR model.

Unit-2: Linear Programming Method

Structure of LPP, Assumptions of LPP, Application areas of LPP, Guidelines for formulation of LPP, Formulation ofLPPFordifferentareas**,**solvingofLPPbyGraphicalMethod:Extremepointmethodand ISO-cost profit method, simplex method, two phase method, Big-M method, converting primal LPP to dual LPP, Limitations of LPP.

Unit-3TransportationProblemandAssignmentModel

Mathematical Model of transportation problem, Methods for finding Initial feasiblesolution: Northwest corner Method, Least Cost Method, Vogel’s approximation Method, Test of optimality by Modi Method, Variation Transportation, and Problems like unbalanced Supply and demand, Degeneracy and its resolution. Algorithm for solving assignment model, Hungarians Method for solving assignment problem, variation so assignment problem:
Multiple Optimal Solutions,
Maximization case in assignment problem, unbalanced assignment problem, travelling salesman problem.

Unit-4 Game Theory and Decision Theory

Introduction – Two Person Zero-Sum Games, Pure Strategies, Games with Saddle Point, Mixed strategies, Rules of Dominance, Solution Methods of Games without Saddle point – Algebraic, matrix and arithmetic methods. Introduction to Decision Theory, ingredients of decision problems. Decision making – under uncertainty, cost of uncertainty, under risk, under perfect information, decision tree, construction of decision tree

Unit-5: PERT & CPM

Drawing networks–identifying critical path–probability of completing the project within given time-project crashing–optimum cost and optimum duration.

UNIT-I

Introduction to Operations Research

Nature and Scope of Operations Research:

Modern technological advance is accompanied by a growth of scientific techniques. While existing methods have been improved to meet the challenge of problems arising from the development of commerce and industry, a large number of new techniques or so-called sophisticated tools of analysis have been and are being devised to enlarge the extent of scientific knowledge to unlimited bounds of its applications. Such techniques have brought about a virtual revolution and can be reckoned as controlling forces in different walks of life.

Operational Research, or simply OR, originated in the context of military operations, but today it is widely accepted as a powerful tool for planning and decision-making, especially in business and industry. The OR approach has provided a new tool for managing conventional management problems. In fact, operational research techniques do constitute a scientific methodology of analyzing the problems of the business world. They provide an improved basis for taking management decisions. The practice of OR helps in tackling intricate and complex problems such as that of resource allocation, product mix, inventory management, sequencing and scheduling, replacement, and a host of similar problems of modern business and industry.

Definitions:

‗OR is the application of scientific methods, techniques and tools to problems involving operations of systems so as to provide those in control of operations with optimum solutions to the problems'.

- *Churchman, Ackoff andArnoff*

‗Operations Research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control'.

- *P.M. Morse and G.F.Kimball*

‗OperationsResearchisapplieddecisiontheory.Itusesanyscientific,mathematicalorlogicalmeans to attempt to cope with the problems that confront the executive when he tries to achieve a thorough going rationality in dealing with his decisionproblems'.

- *D.W. Miller and M.K. Starr*

Origins of Operations Research:

The subject of Operations Research (OR) was developed in military context during World War II, pioneered by the British scientists. At that time, the military management in England appointed a study group of scientists to deal with die strategic and tactical problems related to air and land defence of the country. The main reason for conducting the study was that they were having very limited military resources. It was, therefore, necessary to decide upon the most effective way of utilizing these resources. As the name implies, operations research was apparently invented because the team was dealing with research on (military) operations. The scientists studied the various problems and on the basis of quantitative study of operations suggested certain approaches which showed remarkable success.

The encouraging results obtained by the British operations research teams consisting of personnel drawn from various fields like mathematics, physics, biology, psychology and other physical sciences, quickly motivated the United States military management to start similar activities. Successful innovations of the US teams included the development of new flight patterns, planning sea mining and effective utilization of electronic equipment. Similar OR teams also started functioning in Canada and France. These OR teams were usually assigned to the executive-in-charge of operations and as such their work came to be known as Operational Research in the UK and by a variety of names in the United States: operational analysis, operations evaluation, operations research, systems analysis, systems evaluation and systems research. The name operational research ‘oroperationsresearch‘orsimplyORismostwidelyusednowadaysalloverthe world,for the new approach to systematic and scientific study of the operations of the system. Till the 1950s, use of operations research was mainly confined to military purposes.

After the end of World War II, the success of military teams attracted the attention of industrial managers who were seeking solutions to their complex managerial problems. At the end of the War, expenditures on defence research were reduced in the UK and this led to the release of many OR workers from the military at a time when industrial managers were confronted with the need to reconstruct most of Britain‘s manufacturing industries and plants that had been damaged in War. Executives in such industries sought assistance from the said OR workers. But in the USA, most war experienced operations research workers remained in military service as the defence research was increased and consequently, operations research was expanded at the end of the War. It was only in the early 1950s, that industry in the USA began to absorb the operations research workers under the pressure for increased demands for greater productivity originated because of the outbreak of the Korean conflict and because of technological developments in industry! Thus, operations research began to develop in industrial field in the United States since the year 1950. The Operations Research Society of America was formed in 1953 and the International Federation of Operations Research was formed in1957.

Societies were established. Various journals relating to operations research began to appear in different countries in the years that followed the mid-1950s. Courses and curricula in operations research in different universities and other academic institutions began to proliferate in the United States. Other countries rapidly followed suit and thus, Operations Research came to be applied for solving business and industrial problems. Introduction of Electronic Data Processing (EDP) methods further enlarged the scope for application of OR techniques. With the help of a digited computer many complex problems can be studied on a day-to-day basis. As a result, many industrial concerns are adopting OR as an integrated decision-making tool for their routine decision procedures.

Applications of Operations Research in different

Managerial Areas:

Accounting	**Construction**
• Assigning audit teams effectively • Credit policy analysis • Cash flow planning • Developing standard costs • Establishing costs for by products • Planning of delinquent accountstrategy	• Project scheduling, monitoring and control • Determination of proper workforce • Deployment of workforce • Allocation of resources to projects
Facilities Planning	**Finance**
• Factory location and size decision • Estimation of number of facilities required • Hospital planning • International logistic system design • Transportation loading and unloading • Warehouse locationdecision	• Building cash management models • Allocating capital among various alternatives • Building financial planning models • Investment analysis • Port folio analysis • Dividend policymaking
Manufacturing	**Marketing**
• Inventory control • Marketing balance projection • Production scheduling • Production smoothing	• Advertising budget allocation • Product introduction timing • Selection of Product mix • Deciding most effective packaging alternative
OB / Human Resources	**Purchasing**
• Personnel planning • Recruitment of employees • Skill balancing • Training program scheduling • Designing organizational structure more effectively	• Optimal buying • Optimal reordering • Materials transfer **Research and Development** • R & D Projects control • R & D Budget allocation • Planning of Productintroduction

Problem solving and decision making:

1. **OR techniques help the directing authority** in optimum allocation of various limited resources viz., men, machines, money, material, time, etc., to different competing opportunities on an objective basis for achieving effectively the goal of a business unit. They help the chief of executive in broadening the management vision and perspectives in the choice of alternative strategies to the decision problems such as forecasting manpower, production capacities and

capital requirements and plans for their acquisition.

2. **OR is useful to the production management** in
   (i) Selecting the building site for a plant, scheduling and controlling its development and designing its layout;
   (ii) Locating within the plant and controlling the movements of required production materials and finished goods inventories;
   (iii) Scheduling and sequencing production by adequate preventive maintenance with optimum number of operatives by proper allocation of machines; and
   (iv) Calculating the optimum product-mix.
3. **OR is useful to personnel management** to find out
   (i) The optimum manpower planning;
   (ii) The number of persons to be maintained on permanent or full time roll;
   (iii) The number of persons to be kept in a work pool intended for meeting the absenteeism;
   (iv) The optimum manner of sequencing and routing of personnel to a variety of jobs; and
   (v) In studying personnel recruiting procedures, accident rates and labour turnover.
4. **OR techniques equally help marketing management** to determine
   (i) Where distribution points and warehousing should be located, their size, quantity to be stocked and the choice of customers;
   (ii) The optimum allocation of sales budget to direct selling and promotional expenses;
   (iii) The choice of different media of advertising and bidding strategies; and
   (iv) The consumer preferences relating to size, colour, and packaging, for various products as well as to outbid and out with competitors.
5. **OR is also very useful to financial management** in
   (i) Finding long-range capital requirements as well as how to generate these requirements;
   (ii) Determining optimum replacement policies;
   (iii) Working out a profit plan for the firm;
   (iv) Developing capital-investments plans; and
   (v) Estimating credit and investment risks.

Quantitative and qualitative analysis:

OR—though it is a great aid to management as explained earlier cannot be a substitute for decision-making. The choice of a criterion as to what is actually best for a business enterprise is still that of an executive who has to fall back upon his experience and judgment. This is so because of the several limitations of OR.

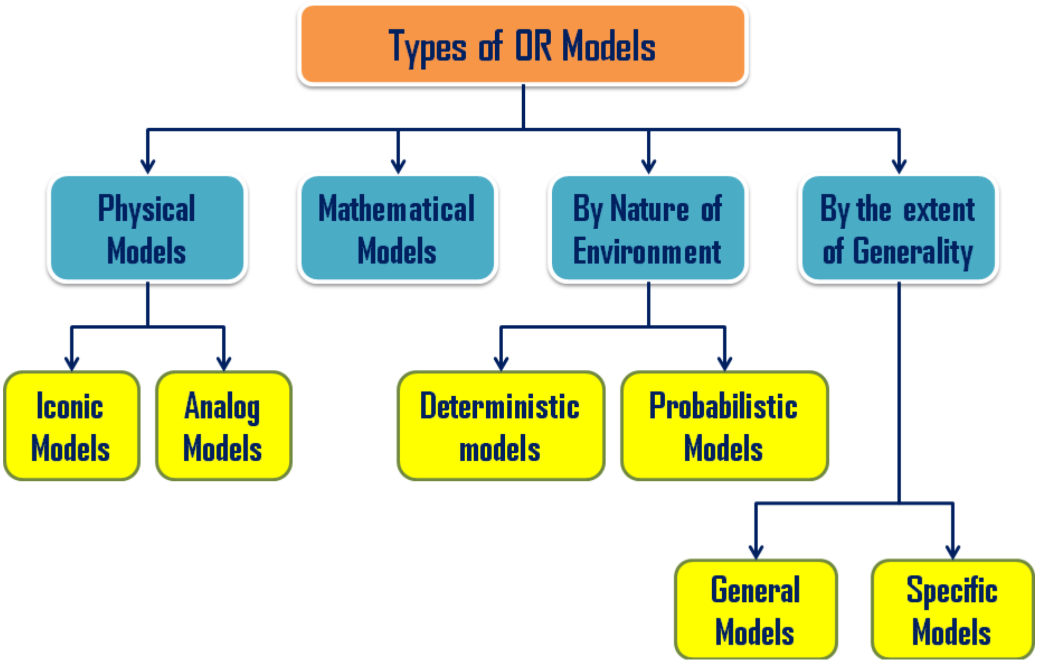
Some important limitations are as follows:

1. **The inherent limitations concerning mathematical expressions.**
   OR involves the use of mathematical models, equations and similar other mathematical expressions. Assumptions are always incorporate Re named in the derivation of an equation or model and such an equation or model may be correctly used for the solution of the business problems when the underlying assumptions and variables in the model are present in the concerning problem. If this caution is not given due care, then there always remains the possibility of wrong application of OR techniques. Quite often, operations researchers have been accused of having many solutions without being able to find problems that fit.
2. **High costs are incurred in the use of OR techniques.**
   OR techniques usually prove to be expensive. Services of specialized persons are invariably called for (and along with it the use of computers) while using OR techniques. As such only big concerns can think of using such techniques. Even in big business organizations, we can expect that OR techniques will continue to be of limited use simply because they are not in many cases worth their cost. As opposed to this, a typical manager exercising intuition and judgment may be able to make a decision very inexpensively. Thus, the use of OR is a costlier affair and this constitutes an important limitation of OR.
3. **OR does not take into consideration the intangible factors, i.e., non-measurable human factors.**
   OR makes no allowance for intangible factors such as skill, attitude, and vigor of the management people in taking decisions but in many instances success or failure hinges upon the consideration of such non-measurable intangible factors. There cannot be any magic formula for getting an answer to management problems; much depends upon proper managerial attitudes and policies.
4. **OR is only a tool of analysis and not the complete decision-making process.**
   It should always be kept in mind that OR alone cannot make the final decision. It is just a tool and simply suggests best alternatives, but in the final analysis many business decisions will involve human element. Thus, OR is at best a supplement rather than a substitute for management; subjective judgment is likely to remain a principal approach to decision-making.
5. **Other limitations.** Among other limitations of OR, the following deserve mention:
   - (i) **Bias.** The operational researchers must be unbiased. An attempt to show results into a confirmation of management‘s prior preferences can greatly increase the likelihood of failure.
   - (ii) **Inadequate objective functions.** The use of a single objective function is often an insufficient basis for decisions. Laws, regulations, public relations, market strategies, etc., may all serve to overrule a choice arrived at in this way.
   - (iii) **Internal resistance.** The implementation of an optimal decision may also confront internal obstacles such as trade unions or individual managers with strong preferences for other ways of doing the job.
   - (iv) **Competence.** Competent OR analysis calls for the careful specification of alternatives, a full comprehension of the underlying mathematical relationships and a huge mass of data. Formulation of an industrial problem to an OR set programme is quite often a difficult task.
   - (v) **Reliability of the prepared solution.** At times, a non-linear relationship is changed into linear for fitting the problem to the linear programming pattern. This may disturb the solution.

Defining a model & types of model:

A model in OR is a simplified representation of an operation, or is a process in which only the basic aspects or the most important features of a typical problem under investigation are considered. The objective of a model is to identify significant factors and interrelationships. The reliability of the solution obtained from a model depends on the validity of the model representing the real system.
A good model must possess the following characteristics:

- It should be capable of taking into account, new formulation without having any changes in its frame.
- Assumptions made in the model should be as small as possible.
- Variables used in the model must be less in number ensuring that it is simple and coherent.
- It should be open to parametric type of treatment.
- It should not take much time in its construction for any problem.

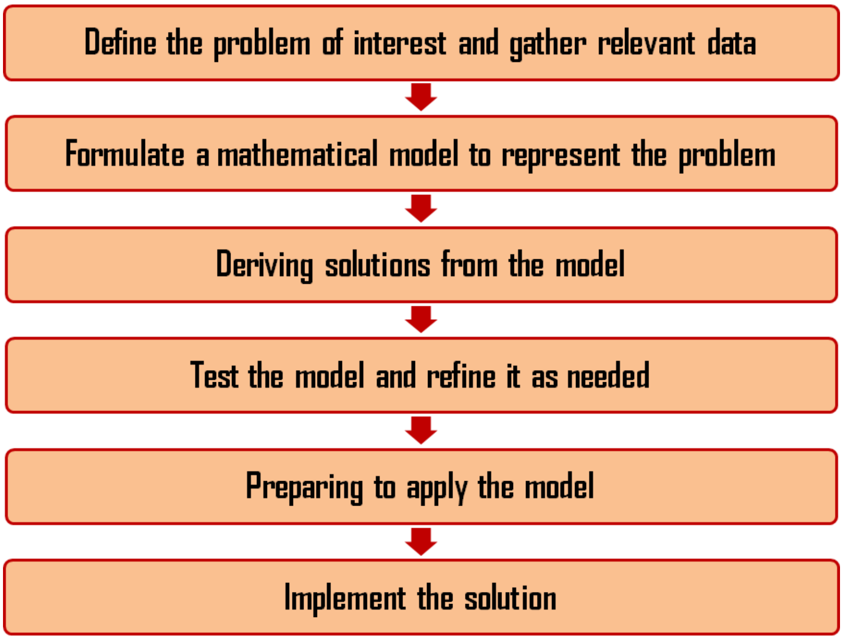


- **Iconic or physical models:** They are pictorial representations of real systems and have the appearance of the real thing. An iconic model is said to be scaled down or scaled up according to the dimensions of the model which may be smaller or greater than that of the real item, e.g., city maps, houses blueprints, globe, and so on. These models are easy to observe and describe, but are difficult to manipulate and are not very useful for the purpose of prediction.
- **Analog models:** These are more abstract than the iconic ones for there is no look alike correspondence between these models and real life items. The models in which one set of properties is used to represent another set of properties are called analog models. After the problem is solved, the solution is reinterpreted in terms of the original system. These models are less specific, less concrete, but easier to manipulate than iconic models.
- **Mathematic or symbolic models:** They are most abstract in nature. They employ a set of mathematical symbols to represent the components of the real system. These variables are related together by means of mathematical equations to describe the behavior of the system. The solution of the problem is then obtained by applying well developed mathematical techniques to the model.
- **Deterministic models:** They are those in which all parameters and functional relationships are assumed to be known with certainty when the decision is to be made. Linear programming and break-even models are the examples of deterministic models.
- **Probabilistic or stochastic models:** These models are those in which at least one parameter or

decision variable is a random variable. These models reflect to some extent the complexity of the real world and the uncertainty surrounding it.

- **General and Specific Models:** When a model presents a system at some specific time, it is known as a specific model. In these models, if the time factor is not considered, they are termed as static models. An inventory problem of determining economic order quantity for the next period assuming that the demand in planning period would remain same as that of today is an example of static model. Dynamic programming may be considered as an example of dynamic model.

Process for Developing an Operations Research Model:



1. **Define the problem of interest and gather relevantdata:**
O.R. is a research into the operation of a man, machine, organization and must consider the economics of the operation. **In formulating/Defining a problem for O.R. study analysis must be made of the following major components:**
(i) The environment;
(ii) The objectives;
(iii) The decision maker;
(iv) The alternative courses of action and constraints.
Out of the above four components environment is most comprehensive as it provides a setting for the remaining three. The operation researcher shall attend conferences, pay visits, send observation and perform research work thus succeeds in getting sufficient data to formulate the problems.
2. **Formulate a mathematical model to represent the problem:**
Once the project is approved by the management, the next step is to construct a model for the system under study. The operation researcher can now construct the model to show the relations and interrelations between a cause and effect or between an action and a reaction. Now the aim of operation researcher is to develop a model which enables him to forecast the effect of factors crucial to the solution of given problem. The proposed model may be tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied by its performance.
3. **Deriving solutions from the model:**
A solution may be extracted from a model either by conducting experiments on it i.e. by simulation or by mathematical analysis. No model will work appropriately if the data is not

appropriate. Such information may be available from the results of experiments or from hunches based on experience. The date collection can clearly effect the models output significantly. Operation researcher should not assume that once he has defined his objective and model, he has achieved his aim of solving the problem. The required data collection consumes time to prepare if data collection errors are to be minimized.

4. **Test the model and refine it as needed:**
A model is never a perfect representation of reality. But if properly formulated and correctly manipulated, it may be useful in providing/predicting the effect of changes in control variables on overall system effectiveness. The usefulness or utility of a model is checked by finding out how well it predicts the effect of these changes. Such an analyze is usually known as sensitivity analysis. The utility or validity of the solution can be verified by comparing the results obtained without applying the solution with the results obtained when it is used.
5. **Preparing to apply the Model/Controlling the solution:**
This step of an OR study establishes control over the solution by proper feedback of information on variables which might have deviated significantly. As such, the significant changes in the system and its environment must be detected and the solution must accordingly be adjusted. This is particularly true when solutions are rules for repetitive decisions or decisions that extend over time.
6. **Implement:**
Implementing the results constitutes the last step of an OR study. Because the objective of OR is not merely to produce reports but to improve the performance of systems, the results of the research must be implemented, if they are accepted by the decision makers. It is through this step that the ultimate test and evaluation of the research is made and it is in this phase of the study the researcher has the greatest opportunity for learning.

Practices & opportunities in OR:

1. **Linear programming** is used in finding a solution for optimizing a given objective such as profit maximization or cost minimization under certain constraints. This technique is primarily concerned with the optimal allocation of limited resources for optimizing a given function. The name linear programming is because of the fact that the model in such cases consists of linear equations indicating linear relationship between the different variables of the system. Linear programming technique solves product-mix and distribution problems of business and industry. It is a technique used to allocate scarce resources in an optimum manner in problems of scheduling, product-mix, and so on. Key factors under this technique include an objective function, choice among several alternatives, limits or constraints stated in symbols and variables assumed to be linear.
2. **Waiting line or queuing theory** deals with mathematical study of queues. Queues are formed whenever the current demand for service exceeds the current capacity to provide that service. Waiting line technique concerns itself with the random arrival of customers at a service station where the facility is limited. Providing too much of capacity will mean idle time for servers and will lead to waste of money. On the other hand, if the queue becomes long, there will be a cost due to waiting of units in the queue. Waiting line theory, therefore, aims at minimizing the costs of both servicing and waiting. In other words, this technique is used to analyze the feasibility of adding facilities and to assess the amount and cost of waiting time. With its help we can find the optimal capacity to be installed which will lead to a sort of an economic balance between cost of service and cost of waiting.
3. **Inventory control/planning** aims at optimizing inventory levels. Inventory may be defined as a useful idle resource which has economic value, e.g., raw-materials, spare parts, finished products, etc. Inventory planning, in fact, answers the two questions, viz., how much to buy and when to buy? Under this technique, the main emphasis is on minimizing costs associated

with holding inventories, procurement of inventories and shortage of inventories.

4. **Game theory** is used to determine the optimum strategy in a competitive situation. The simplest possible competitive situation is that of two persons playing zero-sum game, i.e., a situation in which two persons are involved and one person wins exactly what the other loses. More complex competitive situations of the real life can also be imagined where game theory can be used to determine the optimum strategy.

5. **Decision theory** concerns with making sound decisions under conditions of certainty, risk and uncertainty. As a matter of fact, there are three different types of states under which decisions are made, viz., deterministic, stochastic and uncertainty and the decision theory explains how to select a suitable strategy to achieve some object or goal under each of these three states.

6. **Network analysis** involves the determination of an optimum sequence of performing certain operations concerning some jobs in order to minimize overall time and/or cost. Programme Evaluation and Review Technique (PERT), Critical Path Method (CPM) and other network techniques such as Gantt Chart comes under Network Analysis. Key concepts under this technique are network of events and activities, resource allocation, time and cost considerations, network paths and critical paths.

7. **Simulation** is a technique of testing a model which resembles a real life situation. This technique is used to imitate an operation prior to actual performance. Two methods of simulation are there: One is Monte Carlo method of simulation and the other is System Simulation Method. The former one using random numbers is used to solve problems which involve conditions of uncertainty and where mathematical formulation is impossible, but in case of System Simulation, there is a reproduction of the operating environment and the systemallowsforanalysingtheresponsefromtheenvironmenttoalternativemanagementactions. This method draws samples from a real population instead of drawing samples from a table of random numbers.

8. **Integrated production models** aims at minimizing cost with respect to workforce, production and inventory. This technique is highly complex and is used only by big business and industrial units. This technique can be used only when sales and costs statistics for a considerable long period are available.

9. **Non-linear programming** is that form of programming in which some or all the variables are curvilinear. In other words, this means that either the objective function or constraints or both are not in the linear form. In most practical situations, we encounter nonlinear programming problems, but for computation purpose we approximate them as linear programming problems. Even then, there may remain some non-linear programming problems which may not be fully solved by presently known methods.

10. **Dynamic programming** refers to the systematic search for optimal solutions to problems that involve many highly complex inter relations that are, moreover, sensitive to multistage effects such as successive time phases.

11. **Heuristic programming** also known as discovery method, refers o step by step search toward an optimum when a problem cannot be expressed in the mathematical programming form. The search procedure examines successively a series of combinations that lead to stepwise improvements in the solution and the search stops when a near optimum has been found.

12. **Integer programming** is a special form of linear programming in which the solution is required in terms of integral numbers (i.e., whole numbers)only.

13. **Algorithmic programming** is just the opposite of heuristic programming. It may also be termed as near mathematical programming. This programming refers to a thorough and exhaustive mathematical approach to investigate all aspects of the given variables in order to obtain optimal solution.

14. **Quadratic programming** refers to a modification of linear programming, in which the objective equations appear in quadratic form, i.e., they contain squared terms.

15. **Parametric programming** is the name given to linear programming when the later is

modified for the purpose of inclusion of several objective equations with varying degrees of priority. The sensitivity of the solution to these variations is then studied.

16. **Probabilistic programming**, also known as stochastic programming, refers to linear programming that includes an evaluation of relative risks and uncertainties in various alternatives of choice for management decisions.
17. **Search theory concerns itself with search problems** are characterized by the need for designing a procedure to collect information on the basis of which one or more decisions are made. This theory is useful in places in which some events are known to occur but the exact location is not known. The first search model was developed during the World War II to solve decision problems connected with air patrols and their search for submarines. Advertising agencies‘ search for customers, personnel departments‘ search for good executives are some examples of search theory‘s application in business.
18. **The theory of replacement** is concerned with the prediction of replacement costs and the determination of the most economic replacement policy. There are two types of replacement models - one type of models deal with replacing equipment that deteriorate with time and the other type of models helps in establishing replacement policy for those equipment which fail completely and instantaneously.

Short comings of using an OR model:

1. **Effective Decisions**
   It helps the managers to take better and quicker decisions. It increases the number of alternatives. It helps the managers to evaluate the risk and results of all the alternative decisions.
2. **Better Coordination**
   It helps to coordinate all the decisions of the organisation. It coordinates all the decisions taken by the different levels of management and the various departments of the organisation. For e.g. It coordinates the decisions taken by the production department with the decisions taken by the marketing department.
3. **Facilitates Control:**
   It helps the manager to control his subordinates. It helps the manager to decide which work is most important. The manager does the most important work himself, and he delegates the less important work to his subordinates. It helps a manager to fix standards for all the work. It helps him to measure the performance of the subordinates. It helps the manager to find out and correct the deviations (difference) in the performance.
4. **Improves Productivity:**
   It helps to improve the productivity of the organisation. It helps to decide about the selection, location and size of the factories, warehouses, etc. It helps in inventory control. It helps in production planning and control. It also helps in manpower planning. OR is used in expansion, modernization, installation of technology, etc. OR uses many different mathematical and statistical techniques to improve productivity. Simulation is used by many organizations‘ to improve their productivity. That is, they try out much production improvement techniques on a small scale. If these techniques are successful then they are used on a large scale.

UNIT-II

LINEAR PROGRAMMING PROBLEM METHOD

Structure of LPP:

The development of linear programming has been ranked among the most important scientific advances of the mid-20th century, and we must agree with this assessment. Its impact since just 1950 has been extraordinary. Today it is a standard tool that has saved many thousands or millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world; and its use in other sectors of society has been spreading rapidly. A major proportion of all scientific computation on computers is devoted to the use of linear programming.

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as *constraints*. Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner on the basis of a given criterion of optimality.

Linear Programming is a special and versatile technique which can be applied to a variety of management problems viz. Advertising, Distribution, Investment, Production, Refinery Operations, and Transportation analysis. The linear programming is useful not only in industry and business but also in non-profit sectors such as Education, Government, Hospital, and Libraries. The linear programming method is applicable in problems characterized by the presence of decision variables. The objective function and the constraints can be expressed as linear functions of the decision variables.

The decision variables represent quantities that are, in some sense, controllable inputs to the system being modeled. An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption. There is always some practical limitation on the availability of resources viz. man, material, machine, or time for the system. These constraints are expressed as linear equations involving the decision variables. Solving a linear programming problem means determining actual values of the decision variables that optimize the objective function subject to the limitation imposed by the constraints.

The main important feature of linear programming model is the presence of linearity in the problem. The use of linear programming model arises in a wide variety of applications. Some model may not be strictly linear, but can be made linear by applying appropriate mathematical transformations. Still some applications are not at all linear, but can be effectively approximated by linear models. The ease with which linear programming models can usually be solved makes an attractive means of dealing with otherwise intractable nonlinear models.

Characteristics:

All linear programming problems must have following five characteristics:

(a) **Objective function:** There must be clearly defined objective which can be stated in quantitative way. In business problems the objective is generally profit maximization or cost minimization.

(b) **Constraints:** All constraints (limitations) regarding resources should be fully spelt out in mathematical form.

(c) **Non-negativity:** The value of variables must be zero or positive and not negative. For example, in the case of production, the manager can decide about any particular product number in positive or minimum zero, not the negative.

(d) **Linearity:** The relationships between variables must be linear. Linear means proportional relationship between two _or more variable, i.e., the degree of variables should be maximum one.

(e) **Finiteness:** The number of inputs and outputs need to be finite. In the case of infinite factors, to compute feasible solution is not possible.

Some Important Points

1. A set of values X_1 , X_2 ... X_n which satisfies the constraints (2) of the LPP is called its solution.
2. Any solution to a LPP which satisfies the non-negativity restrictions (3) of the LPP is called its feasible solution.
3. Any feasible solution which optimizes (Minimizes or Maximizes) the objective function (1) of the LPP is called its optimum solution.
4. Given a system of m linear equations with n variables (m < n) any solution which is obtained by solving for m variables keeping the remaining n-m variables zero is called a basic solution. Such m variables are called basic variables and the remaining variables are called non-basic variables.
5. A basic feasible solution is a basic solution which also satisfies (3), that is all basic variables are non-negative.
   Basic feasible solutions are of two types:
   (a) Non-degenerate: A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive X_i (i = 1, 2 m) i.e. None of the basic variables are zero.
   (b) Degenerate: A basic feasible solution is said to degenerate if one or more basic variables are zero.
6. If the value of the objective function Z can be increased or decreased indefinitely such solutions are called unbounded solutions.

Assumptions of Linear Programming Problem:

1. **Proportionality:** The basic assumption underlying the linear programming is that any change in the constraint inequalities will have the proportional change in the objective function. This means, if product contributes Rs 20 towards the profit, then the total contribution would be equal to $20x_1$, where x_1 is the number of units of the product. **For example,** if there are 5 units of the product, then the contribution would be Rs 100 and in the case of 10 units, it would be Rs 200. Thus, if the output (sales) is doubled, the profit would also be doubled.
2. **Additivity:** The assumption of additivity asserts that the total profit of the objective function is determined by the sum of profit contributed by each product separately. Similarly, the total amount of resources used is determined by the sum of resources used by each product separately. This implies, there is no interaction between the decision variables.
3. **Continuity:** Another assumption of linear programming is that the decision variables are continuous. This means a combination of outputs can be used with the fractional values along with the integer values. **For example**, If $5^2/3$ units of product A and $10^1/3$ units of product B to be produced in a week. In this case, the fractional amount of production will be taken as a work-in-progress and the remaining production part is taken in the following week. Therefore, a production of 17 units of product A and 31 units of product B over a three-week period implies $5^2/3$ units of product A and $10^1/3$ units of product B per week.
4. **Certainty:** Another underlying assumption of linear programming is a certainty, i.e. the parameter of objective function coefficients and the coefficients of constraint inequalities is known with certainty. Such as profit per unit of product, availability of material and labor per unit, requirement of material and labor per unit are known and is given in the linear programming problem.

5. **Finite Choices:** This assumption implies that the decision maker has certain choices, and the decision variables assume non-negative values. The non-negative assumption is true in the sense, the output in the production problem cannot be negative. Thus, this assumption is considered feasible.

Thus, while solving for the linear programming problem, these assumptions should be kept in mind such that the best alternative is chosen.

Applications areas of LPP:

LP technique is applied to a wide variety of problems listed below:

(a) Optimizing the product mix when the production line works under certainspecification;
(b) Securing least cost combination of inputs;
(c) Selecting the location of Plant;
(d) Deciding the transportation route;
(e) Utilizing the storage and distribution centers;
(f) Proper production scheduling and inventory control;
(g) Solving the blending problems;
(h) Minimizing the raw materials waste;
(i) Assigning job to specialized personnel.

The fundamental characteristic in all such cases is to find optimum combination of factors after evaluating known constraints. LP provides solution to business managers by understanding the complex problems in clear and sound way.

The basic problem before any manager is to decide the manner in which limited resources can be used for profit maximization and cost minimization. This needs best allocation of limited resources for this purpose linear programming can be used advantageously.

Guidelines for formulation of LPP:

Linear Programming models consist of an objective function and the constraints on that function. A linear programming model takes the following form:

Objective function:

$$Z = a_1X_1 + a_2X_2 + a_3X_3 + \ldots + a_n X_n$$

Constraints:

$$b_{11}X_1 + b_{12}X_2 + b_{13}X_3 + \ldots + b_{1n}X_n < c_1$$
$$b_{21}X_1 + b_{22}X_2 + b_{23}X_3 + \ldots + b_{2n}X_n < c_2$$
$$::::$$
$$\ldots.$$
$$b_{m1}X_1 + b_{m2}X_2 + b_{m3}X_3 + \ldots + b_{mn} X_n < c_m$$

In this system of linear equations, Z is the objective function value that is being optimized, $X_i$are the decision variables whose optimal values are to be found, and ,and $c_i$are constants derived from the specifics of the problem.

Formulation of LPP for different areas:

The procedure for mathematical formulation of an LP problem consists of the following steps:

Step1 Write down the decision variables of the problem.

Step2 Formulate the objective function to be optimized (Maximized or Minimized) as a linear function of the decision variables.

Step3 Formulate the other conditions of the problem such as resource limitation, market constraints and interrelations between variables as linear in equations or equations in terms of the decision variables.

Step4 Add the non-negativity constraint from the considerations so that the negative values of the decision variables do not have any valid physical interpretation.

The objective function, the set of constraint and the non-negative constraint together form a linear programming problem.

The following guidelines help to reduce the risk of errors in problem formulation:

- Be sure to consider any initial conditions.
- Make sure that each variable in the objective function appears at least once in the constraints.
- Consider constraints that might not be specified explicitly. For example, if there are physical quantities that must be non-negative, then these constraints must be included in the formulation.

Solving of LPP by Graphical Method:

A simple linear programming problem with two decision variables can be easily solved by the graphical method.

Procedure for Solving LPP by Graphical Method

The steps involved in the graphical method are as follows.

Step1 Consider each inequality constraint as an equation.

Step2 Plot each equation on the graph as each will geometrically represent a straight line.

Step3 Mark the region. If the inequality constraint corresponding to that line is O then the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality constraint P sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region, thus obtained, is called the feasible region.

Step4 Assign an arbitrary value, say zero, for the objective function.

Step5 Draw a straight line to represent the objective function with the arbitrary value (i.e., a straight line through the origin).

Step6 Stretch the objective function line till the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin, passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passes through at least one corner of the feasible region.

Step7 Find the coordinates of the extreme points selected in step 6 and find the maximum or minimum value of Z.

Note: As the optimal values occur at the corner points of the feasible region, it is enough to calculate the value of the objective function of the corner points of the feasible region and select the one which gives the optimal solution, i.e., in the case of maximization problem the optimal point corresponds to the corner point at which the objective function has a maximum value and in the case of minimization, the corner point which gives the objective function the minimum value is the optimal solution.

General Formulation of LPP

The general formulation of the LPP can be stated as follows: Maximize or Minimize

Z-$C_1X_1 + C_2X_2 + \cdots + C_n X_n$

Subject to _m' constraints

$$a_{11}X_1 + a_{12}X_2 + \cdots + a_{1j} X_j + \cdots + a_m X_n \leq = \geq b_1$$
$$a_{21}X_1 + a_{22}X_2 + \cdots + a_{2j} X_j + \cdots + a_{2n} X_n \leq = \geq b_2$$
$$a_{i1}X_1 + a_{i2}X_2 + \cdots + a_{ij} X_j + \cdots + a_{in} X_n$$
$$\ldots$$
$$a_{m1}X_1 + a_{m2}X_2 + \cdots + a_{mj} X_j + \cdots + a_{mn} X_n \leq = \geq b_m$$

In order to find the values of n decision variables $X_1, X_2, \ldots$,to maximize or minimize the objective function and the non-negativity restrictions

$X_1 \geq 0,\ X_2 \geq 0 \geq X_n \geq 0$

Matrix Form of LPP

The linear programming problem can be expressed in the matrix form as follows:

Maximize or Minimize $Z = CX$

Subject to $\begin{matrix} \leq \\ = \\ \geq \end{matrix} b$

$X \geq 0$

Where x = $\begin{matrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{matrix}$, b = $\begin{matrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{matrix}$, C = $C_1\ C_2 - C_n$

and A= $\begin{matrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & \dots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{matrix}$

Extreme point method:

Step1 Formulate the problem into LP

Step2 Graph the limitations or constraints, initially ignoring the inequalities and decide the feasible region, taking into account the inequality of the relationships. This feasible region should be indicated in the form of a convex polygon.

Step3 Determine the point locations of the extreme points of the feasible region. Step4 Evaluate the value of the objective function at all feasible region.

Step5 Determine the extreme point to obtain the best or optimal value become the value of decision variables from where the value of the function becomes optimal.

Step6 Identify the feasible region and extreme points of this region

Step7 Drawan ISO-profitor ISO-cost line for a particular value of the objectives functions. As the name implies, the cost/profit on all points is the same.

Step8 Move the ISO-cost/profit lines parallel in the direction of increasing/decreasing objective function values.

Step9 The feasible extreme point is then located, for which the solution i.e. where ISO-profit/cost is largest/smallest.

Simplex Method:

The Linear Programming with two variables can be solved graphically. The graphical method of solving linear programming problem is of limited application in the business problems as the number of variables is substantially large. If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method, developed by George Dantzig in 1947; it has proved to be a remarkably efficient method that is used routinely to solve huge problems. The simplex method is an algebraic procedure. However, its underlying concepts are geometric. Understanding these geometric concepts provides a strong intuitive feeling for how the simplex method operates and what makes it so efficient. The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function. The simplex method also helps the decision maker/manager to identify the following:

- Redundant Constraints
- Multiple Solutions
- Unbounded Solution
- Infeasible Problem

It is an iterative procedure for solving LPP in a finite number of steps. This method provides an algorithm which consists of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more as the case may be than at the previous vertex. This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of unbounded solution.

Simplex Algorithm

For the solution of any LPP by simplex algorithm the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

Step 1 Check whether the objective functions of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by

$$\text{Min } Z = -\text{Max } (-Z)$$

Step 2 Check whether all b_i (i = 1,2 … m) are positive. If any one of b_i is negative then multiply the in equation of the constraint by –1 so as to get all b_i to be positive.

Step 3 Express the problem in standard form by introducing slack/surplus variables, to convert the inequality constraints into equations.

Step 4 Obtain an initial basic feasible solution to the problem in the form $X_B = B^{-1}b$ and put it in the first column of the simplex table. Form the initial simplex table as follows

C_B	S_B	C_j X_B	C_1 X_1	C_2 X_2	C_3 X_3	… X_4	… …X_n	0 0….0 S_1..S_2…..S_m
C_{B1}	S_1	b_1	a_{11}	a_{12}	a_{13}	a_{14}	…..a_{1n}	10….0
C_{B2}	S_2	b_2	a_{21}	a_{22}	a_{23}	a_{24}	…..a_{2n}	10….0

Step 5 Compute the net evaluations $Z_j – C_j$ by using the relation $Z_j – C_j = CB (a_j – c_j)$.

Examine the sign of $Z_j – C_j$.

(i) If all $Z_j – C_j \geq 0$, then the initial basic feasible solution X_B is an optimum basic feasible solution.

(ii) If at least one $Z_j – C_j > 0$, then proceed to the next step as the solution is not optimal.

Step 6 (To find the entering variable i.e., key column)

If there are more than one negative $Z_j – C_j$, choose the most negative of them.

Let it be $Z_r – C_r$ for some j = r. This gives the entering variable X_r and is indicated by an arrow at the bottom of the r^{th} column. If there are more than one variable having the same most negative $Z_j – C_j$ then any one of the variable can be selected arbitrarily as the entering variable.

(i) If all $X_{ir} \leq 0$ (i = 1, 2 … m) then there is an unbounded solution to the given problem.

(ii) If at least one $X_{ir} > 0$ (i = 1, 2 … m) then the corresponding vector X_r enters the basis.

Step 7 (To find the leaving variable or key row)

Compute the ratio (X_{Bi} / X_{ir}, $X_{ir} > 0$).

If the minimum of these ratios be X_{Bi} / X_{kr}, then choose the variable X_k to leave the basis called the key row and the element at the intersection of key row and key column is called the key element.

Step 8 Form a new basis by dropping the leaving variable and introducing the entering variable along with the associated value under CB column. Convert the leading element to unity bydividing the key equation by the key element and all other elements in its column to zero by using theformula

$$\text{New Element} = \text{Old Element} - \frac{\text{Product of elements in key row and key column}}{\text{Key element}}$$

Step 9 Go to step (5) and repeat the procedure until either an optimum solution is obtained or there is an indication of unbounded solution.

TWO PHASE METHODS OF PROBLEM SOLVING IN LINEAR PROGRAMMING: FIRST AND SECOND PHASE

In this method, the problem is solved in two phases as given below.

First Phase:

(a) All the terms on R.H.S. should be non negative. If some are -ve then they must be made +ve as explained earlier.
(b) Express constraints in standard form.
(c) Add artificial variables in equality constraints or (>) type constraints.
(d) Form a new objective function W which consisted of the sum of all the artificial variables

$$W = A_1 + A_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + A_m$$

Function (W) is known as infeasibility form.
(e) Function W is to be minimized subject to constraints of original problem and the optimum basic feasible solution is obtained.

Any of the following three cases may arise:

i. Min.W>0andatleastoneartificialvariableappearsincolumn–Basicvariables‖atPositive level. In such case, no feasible solution exists for the original L.P.P. and the procedure is stopped.
ii. Min.W=0andatleastoneartificialvariableappearsincolumn–BasicVariables‖atzerolevel. In such a case, the optimum basic feasible solution to the infeasibility form may or may not be a basic feasible solution to the given (original) L.P.P. To obtain a basic feasible solution, we continue phase I and try to drive all artificial variables out of the basis and then proceed to phase II.
iii. Min.W=0andnoartificialvariableappearsinthecolumn–Basicvariables‖currentsolution‘.In such a case a basic feasible solution to the original L.P.P. has been found. Proceed to phase II.

Second Phase:

Use the optimum basic feasible solution of phase I as a starting solution for the original L.P.P. Using Simplex method make iterations till an optimal basic feasible solution for it is obtained. It may be noted that the new objective function W is always of minimization type regardless of whether the given (original) L.P.P. is of maximization or minimization type.

BIG-M METHOD

The Big M method is a method of solving linear programming problems. It is a variation of the simplex method designed for solving problems typically encompassing "greater-than" constraints as well as "less-than" constraints - where the zero vector is not a feasible solution. The "Big M" refers to a large number associated with the artificial variables, represented by the letter M.

The following steps are involved in solving an LPP using the Big M method.

Step1 Express the problem in the standard form.

Step2 Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type ≥ or =. However, addition of this artificial variable causes violation of the corresponding constraints. Therefore, we would like to get rid of these variables and would not allow them to appear in the final solution. This is achieved by assigning a very large penalty (–M for maximization and M for minimization) in the objective function.

Step3 Solve the modified LPP by the simplex method, until any one of the three cases may arise.

1. If no artificial variable appears in the basis and the optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.
2. If at least one artificial variable in the basis at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution (though degenerated solution).
3. If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied, then the original problem has no feasible solution. The solution satisfies the constraints but does not optimize the objective function, since it contains a very large penalty M and is called pseudo-optimal solution.

Note: While applying the simplex method, whenever an artificial variable happens to leave the basis, we drop that artificial variable and omit all the entries corresponding to its column from the simplex table.

Drawbacks:

- If optimal solution has any artificial variable with non-zero value, original problem is infeasible.
- Four drawbacks of BIG-M method:
  - How large should Mbe?
  - If M is too large, serious numerical difficulties in a computer
  - Big-M method is inferior than 2 phase method
  - Here feasibility is not known until optimality

Converting Primal LPP to dual LPP:

Every LPP (called the primal) is associated with another LPP (called its dual). Either of the problems can be considered as primal with the other as dual.

The importance of the duality concept is due to two main reasons:

i. If the primal contains a large number of constraints and a smaller number of variables, the labour of computation can be considerably reduced by converting it in to the dual problem and then solvingit.
ii. The interpretation of the dual variables from the cost or economic point of view, proves extremely useful in making future decisions in the activities being programmed.

Formulation of Dual Problem

For formulating a dual problem, first we bring the problem in the canonical form. The following changes are used in formulating the dual problem.

1. Change the objective function of maximization in the primal into that of minimization in the dual and vice versa.
2. The number of variables in the primal will be the number of constraints in the dual and vice versa.
3. The cost coefficients C_1, C_2 ... $C_n$in the objective function of the primal will be the RHS constant of the constraints in the dual and vice versa.
4. In forming the constraints for the dual, we consider the transpose of the body matrix of the primal problem.
5. The variables in both problems are non-negative.
6. If the variable in the primal is unrestricted in sign, then the corresponding constraint in the dual will be an equation and vice versa.

Definition of Dual Problem (Primal to Dual)

Let the Primal problem be

Max $Z=C_1x_1+C_2x_2+\ldots+C_nx_n$

Subject to $a_{11}x_1$ $a_{12}x_2$ … $a_{1n}x_n$

$b_1a_{21}x_1$ $a_{22}x_2$ …

$a_{2n}x_n$ b_2

…

…

…

$a_{m1}x_1$ $a_{m2}x_2$ … $a_{mn}x_n$ b_m

$x_1..x_2\ldots x_3$ 0

Dual Problem (to the above Primal)

Min Z1= b1w1+b2w2+…+bm wm

Subject to $a_{11}w_1a_{21}w_2\ldots$ $a_{m1}w_m$

$C_1a_{12}w_1$ $a_{22}w_2$

… $a_{m2}w_m$ C_2

…

…

…

$a_{1n}w_1$ $a_{2n}w_2$ … $a_{mn}w_n$ C_n

$w_1..w_2\ldots w_3$ 0

Limitations of LPP:

1. It is not easy to define a specific objective function.
2. Even if a specific objective function is laid down, it may not be so easy to find out various technological, financial and other constraints which may be operative in pursuing the given objective.
3. Given a specific objective and a set of constraints, it is possible that the constraints may not be directly expressible as linear inequalities.
4. Even if the above problems are surmounted, a major problem is one of estimating relevant values of the various constant coefficients that enter into a linear programming mode, i.e., prices, etc.
5. This technique is based on the assumption of linear relations between inputs and outputs. This means that inputs and outputs can be added, multiplied and divided. But the relations between inputs and outputs are not always linear. In real life, most of the relations are non-linear.
6. This technique assumes perfect competition in product and factor markets. But perfect competition is not a reality.
7. The LP technique is based on the assumption of constant returns. In reality, there are either diminishing or increasing returns which a firm experiences in production.
8. It is a highly mathematical and complicated technique. The solution of a problem with linear programming requires the maximization or minimization of a clearly specified variable. The solutionofalinearprogrammingproblemisalsoarrivedatwithsuchcomplicatedmethodasthe simplex method' which involves a large number of mathematical calculations.
9. Mostly, linear programming models present trial-and-error solutions and it is difficult to find out really optimal solutions to the various economic problems.

UNIT-III

Assignment Model

Algorithm for solving assignment model:

Given n facilities, n jobs and the effectiveness of each facility to each job, here the problem is to assign each facility to one and only one job so that the measure of effectiveness if optimized. Here the optimization means Maximized or Minimized. There are many management problems has a assignment problem structure. For example, the head of the department may have 6 people available for assignment and 6 jobs to fill. Here the head may like to know which job should be assigned to which person so that all tasks can be accomplished in the shortest time possible. Another example a container company may have an empty container in each of the location 1, 2, 3, 4, 5 and requires an empty container in each of the locations 6, 7, 8, 9, 10. It would like to ascertain the assignments of containers to various locations so as to minimize the total distance. The third example here is, a marketing set up by making an estimate of sales performance for different salesmen as well as for different cities one could assign a particular salesman to a particular city with a view to maximize the overallsales.

Note that with n facilities and n jobs there are n! Possible assignments. The simplest way of finding an optimum assignment is to write all the n! Possible arrangements evaluate their total cost and select the assignment with minimum cost. Bust this method leads to a calculation problem of formidable size even when the value of n is moderate. For n=10 the possible number of arrangements is 3268800.

Difference between Transportation Problem and Assignment Problem

Transportation problem	Assignment problem
No. of sources and number of destinations need not be equal. Hence, the cost matrix is not necessarily a square matrix.	Since assignment is done on one-to-one basis, the number of sources and the numberof destinations are equal. Hence, the cost matrix must be a square matrix.
X_{ij}, the quantity to be transported from i^{th} origin to j^{th} destination can take any possible positive values, and satisfies the rim requirements.	X_{ij}, the j^{th} job is to be assigned to the i^{th} person and can take either the value l or 0.
The capacity and the requirement value is equal to a_i and b_j for the i^{th} source and j^{th} destinations (i=1, 2 …m; j=1,2… n)	The capacity and the requirement value isexactly 1 i.e., for each source of each destination the capacity and the requirement value isexactly 1.
The problem is unbalanced if the total supply and the total demand are not equal.	The problem is unbalanced if the cost matrix is not a square matrix.

The structure of the Assignment problem is similar to a transportation problem, is as follows:

Jobs

Workers		J_1	J_2	J_3	J_4	$J_{...}$	J_n
	W_1	C_{11}	C_{12}	C_{13}	C_{14}	...	C_{1n}
	W_2	C_{21}	C_{22}	C_{23}	C_{24}	...	C_{2n}
	W_3	C_{31}	C_{32}	C_{33}	C_{34}	...	C_{3n}
	W_4	C_{41}	C_{42}	C_{43}	C_{44}	...	C_{4n}
	$W_{...}$	...	...	...	...	...	...
	W_n	C_{n1}	C_{n2}	C_{n3}	C_{n4}		C_{nn}

The element C_{ij} represents the measure of effectiveness when i^{th} person is assigned j^{th} job. Assume that the overall measure of effectiveness is to be minimized. The element x_{ij} represents the number of i^{th} individuals assigned to the j^{th} job. Since i^{th} person can be assigned only one job and j^{th} job can be assigned to only one person we have the following

$$X_{i1} + X_{i2} + \ldots\ldots\ldots\ldots + X_{in} = 1, \text{ where } i = 1,2,\ldots\ldots\ldots\ldots,n$$

$$X_{1j} + X_{2j} + \ldots\ldots\ldots\ldots + X_{nj} = 1, \text{ where } j = 1,2,\ldots\ldots\ldots\ldots,n$$

and the objective function is formulated as

$$\text{Minimize } C_{11}X_{11} + C_{12}X_{12} + \ldots\ldots\ldots\ldots + C_{nn}X_{nn}$$

$$X_{ij} \geq 0$$

The assignment problem is actually a special case of the transportation problem where m = n and $a_i = b_j = 1$. However, it may be easily noted that any basic feasible solution of an assignment problem contains (2n – 1) variables of which (n – 1) variables are zero. Because of this high degree of degeneracy the usual computation techniques of a transportation problem become very inefficient. So, hat a separate computation technique is necessary for the assignment problem.

The solution of the assignment problem is based on the following results:

–If a constant is added to every element of a row/column of the cost matrix of an assignment problem the resulting assignment problem has the same optimum solution as the original assignment problem and vice versa‖. This result may be used in two different methods to solve the assignment problem. If in an assignment problem some cost elements C_{ij} are negative, we may have to convert them into an equivalent assignment problem where all the cost elements are non-negative by adding a suitable large constant to the cost elements of the relevant row or column, and then we look for a feasible solution which has zero assignment cost after adding suitable constants to the cost elements of the various rows and columns. Since it has been assumed that all the cost elements are non-negative, this assignment must be optimum. On the basis of this principle a computational technique known as Hungarian Method is developed. The Hungarian Method is discussed as follows.

Hungarians Method for solving assignment problem:

The solution of an assignment problem can be arrived using the Hungarian Method. The steps involved in this method are as follows.

Step 1: Prepare a cost matrix. If the cost matrix is not a square matrix then add a dummy row (column) with zero cost elements.

Step 2: Subtract the minimum element in each row from all the elements of the respective rows.

Step 3: Further modify the resulting matrix by subtracting the minimum element of each column from all the elements of the respective columns. Thus, obtain the modified matrix.

Step 4: Then, draw minimum number of horizontal and vertical lines to cover all zeroes in the resulting matrix. Let the minimum number of lines be N. Now, there are two possible cases.

Case 1: If N = n, where n is the order of matrix, then an optimal assignment can be made. So make the assignment to get the required solution.

Case 2: If N < n, then proceed to step 5.

Step 5: Determine the smallest uncovered element in the matrix (element not covered by N lines). Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines. Thus, the second modified matrix is obtained

Step 6: Repeat step (3) and (4) until we get the case (i) of Step 4.

Step 7: (To make zero assignment) Examine the rows successively until a row-wise exactly single zero is found. Circle (O) this zero to make the assignment. Then mark a cross (×) over all zeroes if lying in the column of the circled zero, showing that they cannot be considered for future assignment. Continue in this manner until all the zeroes have been examined. Repeat the same procedure for the column also.

Step 8: Repeat the step 6 successively until one of the following situation arises

(i) If no unmarked zero is left, then the process ends.
(ii) If there lies more than one of the unmarked zero in any column or row, then circle one of the unmarked zeroes arbitrarily and mark a cross in the cells of remaining zeroes in its row or column. Repeat the process until no unmarked zero is left in the matrix.

Step 9: Thus, exactly one marked circled zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked circled zeroes will give the optimal assignment.

Variations of Assignment Problem

We assumed that the number of persons to be assigned and the number of jobs were same. Such kind of assignment problem is called as balanced assignment problem. Suppose if the number of person is different from the number of jobs then the assignment problem is called as unbalanced.

If the number of jobs is less than the number of persons, some of them can‘t be assigned any job. So that we have to introduce on or more dummy jobs of zero duration to make the unbalanced assignment problem into balanced assignment problem. This balanced assignment problem can be solved by using the Hungarian Method as discussed in the previous section. The persons to whom the dummy jobs are assigned are left out of assignment.

Similarly, if the number of persons is less than number of jobs then we have introduce one or more dummy persons with zero duration to modify the unbalanced into balanced and then the problem is solved using the Hungarian Method. Here the jobs assigned to the dummy persons are left out.

Multiple Optimal Solutions:

An Assignment problem can have more than one optimal solution, which is called multiple optimal solutions. The meaning of multiple optimal solutions is the total cost or total profit will remain same for different sets or combinations of allocations. It means we have the flexibility of assigning different allocations while still maintaining Minimum (Optimal) cost or Maximum (Optimal) profit. We can detect multiple optimal solutions when there are multiple zeroes in any columns or rows in the final (Optimal) table in the Assignment problem.

Unbalanced assignment Problem:

In the previous section we discussed about the balanced transportation problem i.e. the total supply (capacity) at the origins is equal to the total demand (requirement) at the destination. In this section we are going to discuss about the unbalanced transportation problems i.e. when the total supply isnot equal to the total demand, which are called as unbalanced transportation problem.

In the unbalanced transportation problem if the total supply is more than the total demand then we introduce an additional column which will indicate the surplus supply with transportation cost zero.

Similarly, if the total demand is more than the total supply an additional row is introduced in the transportation table which indicates unsatisfied demand with zero transportation cost.

Maximization case in assignment problem:

In this, the objective is to maximize the profit. To solve this, we first convert the given profit matrix into the loss matrix by subtracting all the elements from the highest element of the given profit matrix. For this converted loss matrix, we apply the steps in Hungarian method to get the optimum assignment.

Infeasible Assignment Problem:

Sometimes it is possible a particular person is incapable of performing certain job or a specific job can't be performed on a particular machine. In this case the solution of the problem takes into account of these restrictions so that the infeasible assignment can be avoided. The infeasible assignment can be avoided by assigning a very high cost to the cells where assignments are restricted or prohibited.

Travelling salesman problem:

Assume a salesman has to visit *n* cities. He wishes to start from a particular city, visit each city once and then return to his starting point. His objective is to select the sequence in which the cities are visited in such a way that his total travelling time is minimized. To visit 2 cities (A and B, there is no choice. To visit 3 cities we have 2! Possible routes. For 4 cities we have 3! Possible routes. In general, to visit *n* cities there are $(n - 1)!$ Possible routes.

Mathematical Formulation of TSP

Let C_{ij} be the distance or time or cost of going from city i to city j. the decision variable X_{ij} be 1 if the salesman travels from city i to city j and otherwise 0. The objective is to minimize the travelling time.

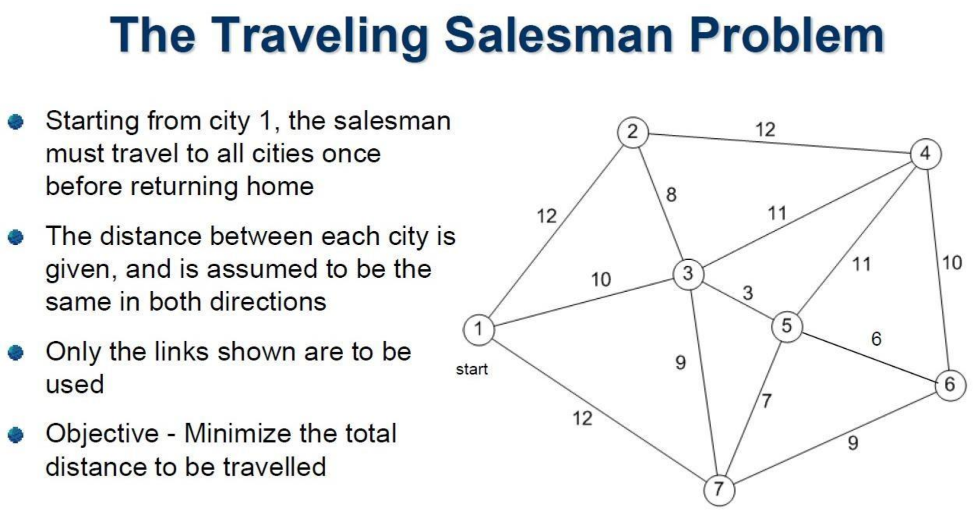
$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \ x_{ij}$$

$$\sum_{j=1}^{n} X_{ij} = 1, \ i = 2 \ldots n.$$

$$\sum_{i=1}^{n} X_{ij} = 1, \ j = 2 \ldots n.$$

and subject to the additional constraint that X_{ij} is so chosen that no city is visited twice before all the cities are completely visited. In particular, going from i directly to j is not permitted. Which means $Cij = \infty$, when $i = j$. In travelling salesman problem we cannot choose the element along the diagonal and this can be avoided by filling the diagonal with infinitely large elements. The travelling salesman problem is very similar to the assignment problem except that in the former care, there is an additional restriction that X_{ij} is so chosen that no city is visited twice before the tour of all the cities is completed.

Solution Treat the problem as an assignment problem and solve it using the same procedures. If the optimal solution of the assignment problem satisfies the additional constraint, then it is also an optimal solution of the given travelling salesman problem. If the solution to the assignment problem does not satisfy the additional restriction then after solving the problem by assignment technique we use the method of enumeration.



Transportation Problem

Mathematical Model of transportation problem:

A special class of linear programming problem is Transportation Problem, where the objective is to minimize the cost of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. warehouses) while satisfying both the supply limits and the demand requirement. Because of the special structure of the Transportation Problem the Simplex Method of solving is unsuitable for the Transportation Problem. The model assumes that the distributing cost on a given rout is directly proportional to the number of units distributed on that route. Generally, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling, and personnel assignment.

Assumptions

1. Only a single type of commodity is being shipped from an origin to a destination.
2. Total supply is equal to the total demand. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, a_i (supply) and b_j (demand) are all positive integers.
3. The unit transportation cost of the item from all sources to destinations is certainly and preciously known.
4. The objective is to minimize the total cost.

Mathematical Formulation

Consider a transportation problem with m origins (rows) and n destinations (columns). Let C_{ij} be the cost of transporting one unit of the product from the i^{th}origin to j^{th} destination. The quantity a_i of commodity available at origin i, the quantity b_j of commodity needed at destination j. The quantity x_{ij} transported from i^{th} origin to j^{th} destination. This following transportation problem can be stated in the tabular form.

	Destinations					
Origins	1	2	3	…	n	Capacity
1	C_{11} x_{11}	C_{12} x_{12}	C_{13} x_{13}	…	C_{1n} x_{1n}	a_1
2	C_{21} x_{21}	C_{22} x_{22}	C_{23} x_{23}	…	C_{2n} x_{2n}	a_2
3	C_{31} x_{31}	C_{32} x_{32}	C_{33}	…	C_{3n} x_{3n}	a_3
m	C_{m1} x_{m1}	C_{m2} x_{m2}	C_{m3} x_{m3}	…	C_{mn} x_{mn}	a_m
Demand	b_1	b_2	b_3	…	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

The linear programming model representing the transportation problem is given by

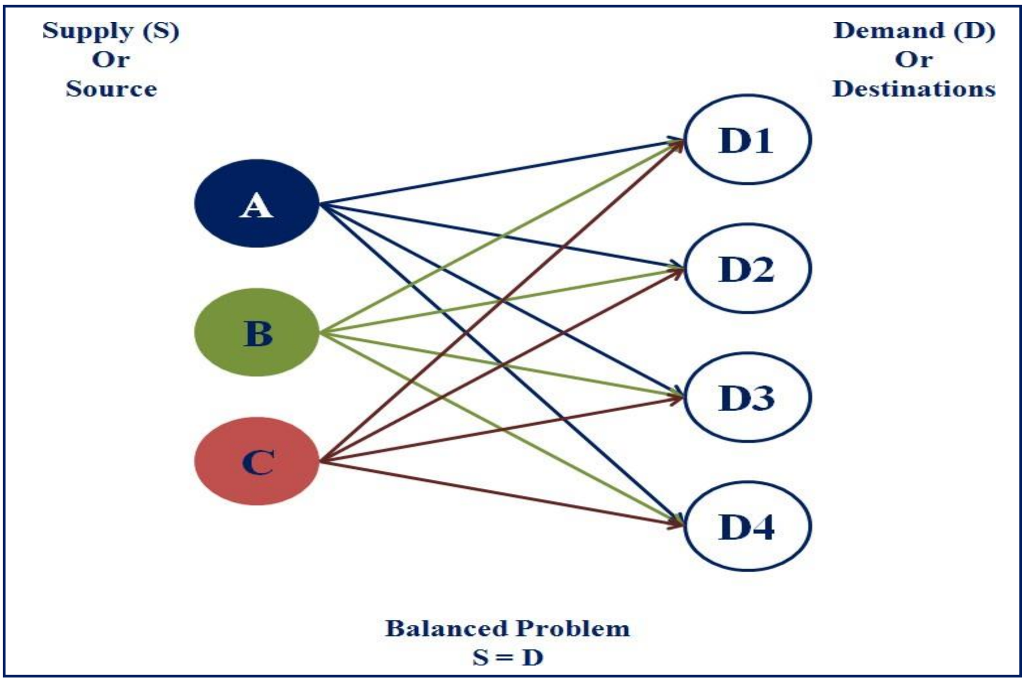
$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

The given transportation problem is said to be balanced if

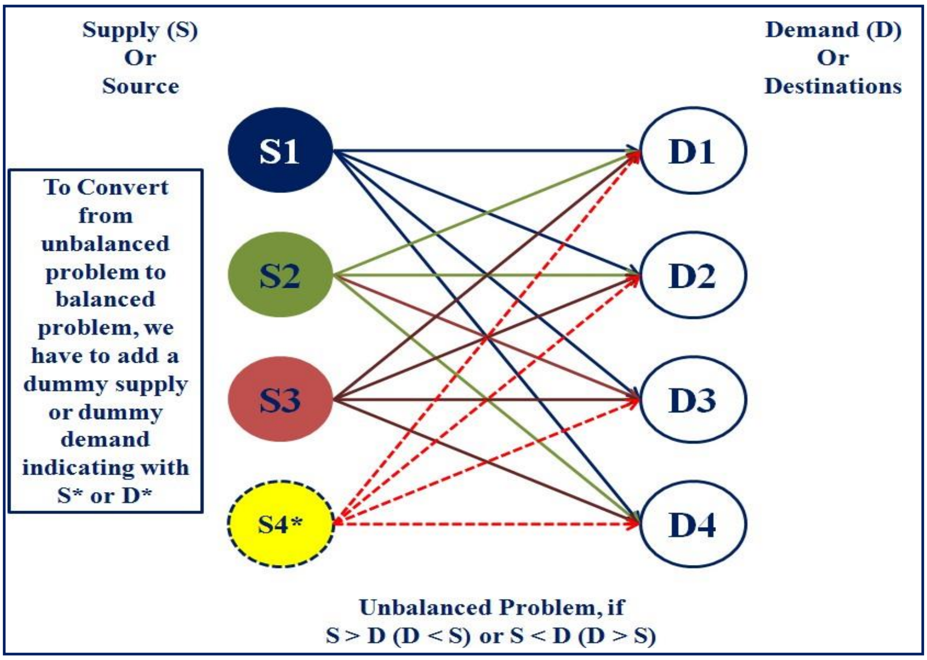
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} bj$$

i.e. if the total supply is equal to the total demand.



Feasible Solution: Any set of non-negative allocations (X_{ij}> 0) which satisfies the row and column sum (rim requirement) is called a feasible solution.

Basic Feasible Solution: A feasible solution is called a basic feasible solution if the number of non-negative allocations is equal to m+n-1, where m is the number of rows and n the number of columns in a transportation table.



Non-degenerate: Basic Feasible Solution Any feasible solution to a transportation problem containing m origins and n destinations is said to be non-degenerate, if it contains m+n-1 occupied cells and each allocation is in independent positions.

- The allocations are said to be in independent positions, if it is impossible to form a closedpath.
- Closed path means by allowing horizontal and vertical lines and all the corner cells areoccupied.

The allocations in the following tables are not in independent positions.

	*	*
	*	*

*			*
*			*

	*			*
			*	*
	*			
			*	
	*		*	

The allocations in the following tables are in independent positions.

	*	*
*		*

*			*
	*		
			*

	*			*
			*	*
	*			
			*	
*			*	

Degenerate Basic Feasible Solution If a basic feasible solution contains less than m+n-1 non-negative allocations, it is said to be degenerate.

Methods for finding Initial feasible solution

Optimal solution is a feasible solution (not necessarily basic) which minimizes the total cost. The solution of a transportation problem can be obtained in two stages, namely initial solution and optimum solution. Initial solution can be obtained by using any one of the three methods, viz,

(i) North west corner rule(NWCR)
(ii) Least cost method or matrix minimum method
(iii) Vogel's approximation method(VAM)

Northwest corner Method:

The method starts at the North West (upper left) corner cell of the tableau (variable X_{11})

Step 1: Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.

Step 2: Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).

Step 3: If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step 1.

Least Cost Method:

The least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which Cij is minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered row. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

Vogels approximation Method:

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: For each row (column) with strictly positive capacity (requirement), determine a penalty by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row(column).

Step 2: Identify the row or column with the largest penalty among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.

Step 3: Select X_{ij} as a basic variable if C_{ij} is the minimum cost in the row or column with largest penalty. Choose the numerical value of X_{ij} as high as possible subject to the row and the column constraints. Depending upon whether a_i or b_j is the smaller of the two i^{th} row or j^{th} column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

Test of optimality by Modi Method :

The Modified Distribution Method, also known as MODI method or u-v method, which provides a minimum cost solution (optimal solution) to the transportation problem. The following are the steps involved in this method.

Step 1: Find the initial basic feasible solution of a TP by using any one of the three methods.

Step 2: Find out a set of numbers u_i and v_j for each row and column satisfying $u_i v_j \backslash C_{ij}$ for each occupied cell. To start with, we assign a number ‗0‘ to any row of column having maximum number of allocations. If this maximum number of allocations is more than one, choose any one arbitrarily.

Step 3: For each empty (unoccupied) cell, we find the sum u_i and v_j written in the bottom left corner of that cell.

Step 4: Find out for each empty cell the net evaluation value $\Delta_{ij} = C_{ij} - (u_i+v_j)$ and which is written at the bottom right corner of that cell. This step gives the optimality conclusion,

(i) If all $\Delta_{ij} > 0$ (i.e., all the net evaluation value) the solution is optimum and a unique solution exists.

(ii) If Δ_{ij} P 0 then the solution is optimum, but an alternate solution exists.

(i) (iii) If at least one $\Delta_{ij} < 0$, the solution is not optimum. In this case, we go to the next step, to improve the total transportation cost.

Step 5: Select the empty cell having the most negative value of Δ_{ij}. From this cell we draw a closed path by drawing horizontal and vertical lines with the corner cells occupied. Assign sign and alternately and find the minimum allocation from the cell having negative sign. This allocation should be added to the allocation having positive sign and subtracted from the allocation having negative sign.

Step 6: The previous step yields a better solution by making one (or more) occupied cell as empty and one empty cell as occupied. For this new set of basic feasible allocations, repeat from the step (2) till an optimum basic feasible solution is obtained.

Unbalanced Supply and Demand:

Definition: The Stepping Stone Method is used to check the optimality of the initial feasible solution determined by using any of the method Viz. North-West Corner, Least Cost Method or Vogel‘s Approximation Method. Thus, the stepping stone method is a procedure for finding the potential of any non-basic variables (empty cells) in terms of the objective function.

Through stepping stone method, we determine that what effect on the transportation cost would be in case one unit is assigned to the empty cell. With the help of this method, we come to know whether the solution is optimal or not.

The series of steps are involved in checking the optimality of the initial feasible solution using the stepping stone method:

1. The prerequisite condition to solve for the optimality is to ensure that the number of occupied cells is exactly equal to m+n-1, where ‗m‘ is the number of rows, while ‗n‘ is equal to the number of columns.
2. Firstly, the empty cell is selected and then the closed path is created which starts from the unoccupiedcellandreturnstothesameunoccupiedcell,calledasa–closedloop‖.Forcreatinga closed loop the following conditions should be kept in mind:
   - In a closed loop, cells are selected in a sequence such that one cell is unused/unoccupied, and all other cells are used/occupied.
   - A pair of Consecutive used cells lies either in the same row or the same column.
   - No three consecutive occupied cells can either be in the same row or column.

- The first and last cell in the closed loop lies either in the same row or column.
- Only horizontal and vertical movement is allowed.

3. Oncetheloopiscreated,assign–+‖or—–signalternativelyoneachcornercelloftheloop,but begin with the–+‖ sign for the un occupied cell.
4. Repeat these steps again until all the unoccupied cells get evaluated.
5. Now, if all the computed changes are positive or are equal to or greater than zero, then the optimal solution has been reached.
6. But in case, if any, value comes to be negative, then there is a scope to reduce the transportation cost further. Then, select that unoccupied cell which has the most negative change and assign as many units as possible. Subtract the unit that added to the unoccupied cell from the other cells with a negative sign in a loop, to balance the demand and supply requirements.

Degeneracy and its resolution:

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than (m + n – 1) positive X_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

2. At the initial solution
3. During the testing of the optimal solution

To resolve degeneracy, we make use of an artificial quantity (d). The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

UNIT-IV

Decision Theory

Introduction:

Decision-making is an everyday process in life. It is the major job of a manager too. A decision taken by a manager has far-reaching consequences on the business. Right decisions will have a salutary effect and wrong ones may prove to be disastrous. Decisions may be classified into two categories, tactical and strategic. Tactical decisions are those which affect the business in the short run. Strategic decisions are those which have a long-term effect on the course of business. These days, in every organization - large or small, the person at the top has to take the crucial decisions, knowing that certain events beyond his control may occur and make him regret the decision. He is uncertain as to whether or not these unfortunate events will happen. In such situations the best possible decision can be made with the use of statistical tools which try to minimize the degree to which the person is likely to regret the decision he takes for a particular problem.

The problem under study may be represented by a model in terms of the following elements:

(i) **The decision-maker:** The decision-maker is charged with the responsibility of taking a decision; he has to select one from a set of possible courses of action.

(ii) **Acts:** These are the alternative courses of action or strategies that are available to the decision-maker. The decision involves a selection among two or more alternative courses of action. The problem is to choose the best of these alternatives to achieve an objective.

(iii) **Events:** These are the occurrences which affect the achievement of the objectives. They are also called states of nature or outcomes. The events constitute a mutually exclusive and exhaustive set of outcomes which describe the possible behavior of the environment in which the decision is made. The decision-maker has no control over which event take place and can will only attach a subjective probability of occurrence of each.

(iv) **Pay-off table:** It represents the economics of a problem, i.e., the revenue and costs associated with any action with a particular outcome. It is an ordered statement of profit or costs resulting under a given situation. A pay-off can be interpreted as the outcome in quantitative form if the decision-maker adopts a particular strategy under a particular state of nature.

(v) **Opportunity loss table:** An opportunity loss is the loss incurred because of the failure to take the best possible action. Opportunity losses are calculated separately for each state of nature that might occur. Given the occurrence of a specific state of nature, we can determine the best possible act. For a given state of nature, the opportunity loss of an act is the difference between the pay-off of that act and the pay-off for the best act that could have been selected.

Ingredients of decision problems :

Decisions often must be made in environments that are much more fraught with uncertainty. Here are a few examples.

1. A manufacturer introducing a new product into the marketplace. What will be the reaction of potential customers? How much should be produced? Should the product be test marketed in a small region before deciding upon full distribution? How much advertising is needed to launch the product successfully?
2. A financial firm investing in securities. Which are the market sectors and individual securities with the best prospects? Where is the economy headed? How about interest rates? How should these factors affect the investment decisions?
3. A government contractor bidding on a new contract. What will be the actual costs of the project? Which other companies might be bidding? What are their likely bids?
4. An agricultural firm selecting the mix of crops and livestock for the upcoming season. What will be the weather conditions? Where are prices headed? What will costs be?

5. An oil company deciding whether to drill for oil in a particular location. How likely is oil there? How much? How deep will they need to drill? Should geologists investigate the site further before drilling?

Decision Making:

In any decision problem, the decision-maker is concerned with choosing from among the available alternative courses of action, the one that yields the best result. If the consequences of each choice are known with certainty, the decision-maker can easily make decisions. But in most of real life problems, the decision-maker has to deal with situations where uncertainty of the outcomes prevails.

The decision-making problems can be discussed under the following heads on the basis of their environments:

1. Decision-making under certainty
2. Decision-making under uncertainty
3. Decision-making under risk

1. Decision-Making under Certainty

In this case, the decision-maker knows with certainty the consequences of every alternative or decision choice. The decision-maker presumes that only one state of nature is relevant for his purpose. He identifies this state of nature, takes it for granted and presumes complete knowledge as to its occurrence.

2. Decision-Making under Uncertainty

When the decision-maker faces multiple states of nature but he has no means to arrive at probability values to the likelihood of occurrence of these states of nature, the problem is a decision problem under uncertainty. Such situations arise when a new product is introduced in the market or a new plant is set up. In business, there are many problems of this _nature‘. Here, the choice of decision largely depends on the personality of the decision-maker.

The following choices are available before the decision-maker in situations of uncertainty.

(a) Maximax Criterion
(b) Minimax Criterion
(c) Maximin Criterion
(d) Laplace Criterion (Criterion of equally likelihood)
(e) Hurwicz Alpha Criterion (Criterion of Realism)

(a) **The maximax decision criterion (criterion of optimism):** The term _maximax‘ is an abbreviation of the phrase maximum of the maximums, and an adventurous and aggressive decision-maker may choose to take the action that would result in the maximum pay-offpossible.

(b) **The minimax decision criterion:** Minimax is just the opposite of maximax. Application of the minimax criteria requires a table of losses instead of gains. The losses are the costs to be incurred or the damages to be suffered for each of the alternative actions and states of nature. The minimaxruleminimizesthemaximumpossiblelossforeachcourseofaction.Theterm _minimax‘ is an abbreviation of the phrase minimum of the maximum. Under each of the various actions, there is a maximum loss and the action that is associated with the minimum of the various maximum losses is the action to be taken according to the minimax criterion.

(c) **The maximin decision criterion (criterion of pessimism):** The maximin criterion of decision-making stands for choice between alternative courses of action assuming pessimistic view of nature. Taking each act in turn, we note the worst possible results in terms of pay-off and select the act which maximizes the minimum pay-off.

(d) **Laplace criterion:** As the decision-maker has no information about the probability of occurrence of various events, he makes a simple assumption that each probability is equally likely. The expected pay-off is worked out on the basis of these probabilities.

(e) **Harwicz alpha criterion:** This method is a combination of maximum criterion and maximax criterion. In this method, the decision maker's degree of optimism is represented by α ,the coefficient of optimism. α varies between 0 and 1. When α = 0, there is total pessimism and when α = 1, there is total optimism.

3. **Decision-Making underRisk**

In this situation, the decision-maker has to face several states of nature. But he has some knowledge or experience which will enable him to assign probability to the occurrence of each state of nature. The objective is to optimize the expected profit, or to minimize the opportunity loss.

For decision problems under risk, the most popular methods used are Expected Monetary Value (EMV) criterion, Expected Opportunity Loss (EOL) criterion or Expected Value of Perfect Information (EVPI).

(a) **EMV Criterion:** When probabilities can be assigned to the various states of nature, it is possible to calculate the statistical expectation of gain for each course of action. The conditional value of each event in the pay-off table is multiplied by its probability and the product is summed up. The resulting number is the EMV for the act. The decision-maker then selects from the available alternative actions, the action that leads to the maximum expected gain (that is the action with highest EMV).

(b) **EOL Criterion:** The difference between the greater pay-off and the actual pay-off is known as opportunity loss. Under this criterion, the strategy which has minimum Expected Opportunity Loss (EOL) is chosen. The calculation of EOL is similar to that of EMV.

(c) **EVPI Method:** The expected value of perfect information (EVPI) is the average (expected) return in the long run, if we have perfect information before a decision is to be made. In order to calculate EVPI, we choose the best alternative with the probability of their state of nature. EVPI is the expected outcome with perfect information minus the outcome with max EMV.

∴ EVPI = Expected value with perfect information – max. EMV

Under Perfect Information:

Decisions often must be made in environments that are much more fraught with uncertainty. Hereare a few examples.

6. A manufacturer introducing a new product into the marketplace. What will be the reaction of potential customers? How much should be produced? Should the product be test marketed in a small region before deciding upon full distribution? How much advertising is needed to launch the product successfully?
7. A financial firm investing in securities. Which are the market sectors and individual securities with the best prospects? Where is the economy headed? How about interest rates? How should these factors affect the investment decisions?
8. A government contractor bidding on a new contract. What will be the actual costs of the project? Which other companies might be bidding? What are their likely bids?
9. An agricultural firm selecting the mix of crops and livestock for the upcoming season. What will be the weather conditions? Where are prices headed? What will costs be?
10. An oil company deciding whether to drill for oil in a particular location. How likely is oil there? How much? How deep will they need to drill? Should geologists investigate the site further before drilling?

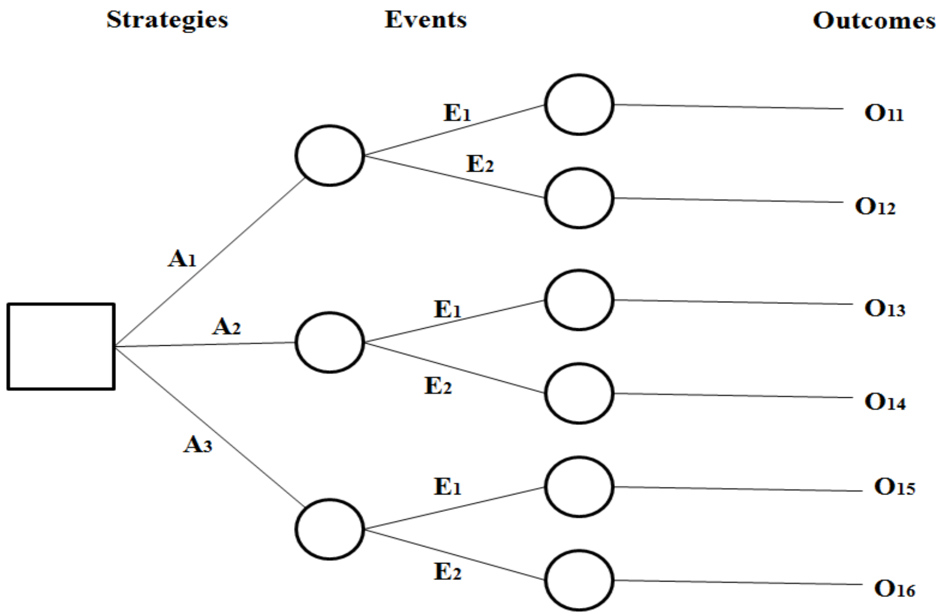
Decision tree & construction of decision tree:

A Decision tree is one of the tools used for the diagrammatic presentation of the sequential and multidimensional aspects of a particular decision problem for systematic analysis and evaluation. Here, the decision problem, the alternative courses of action, the states of nature and the likely outcomes of alternatives are diagrammatically or graphically depicted as branches and sub-branches of a horizontal tree.

The decision tree consists of nodes and branches. The nodes are of two types, decision nodes and chance nodes. Courses of action (or strategies) originate from the decision nodes as the main branches. At the terminal of each main branch there are chance nodes. From these chance nodes, chance events emanate in the form of sub-branches. The respective pay-offs and probabilities associated with alternative courses and chances events are shown along the sub-branches. At the terminal of the sub-branches are shown the expected values of the outcome.

A decision tree is highly useful to a decision-maker in multistage situations which involve a series of decisions each dependent on the preceding one. Working backward, from the future to the present, we are able to eliminate unprofitable branches and determine optimum decisions. The decision tree analysis allows one to understand, simply by inspection, various assumptions and alternatives in a graphic form which is much easier to understand than the abstract analytical form.

The advantages of the decision tree structure are that complex managerial problems and decisions of a chain-like nature can be systematically and explicitly defined and evaluated.



NETWORK ANALYSIS

Project Management

A project is a well defined task which has a definable beginning and a definable end and requires one or more resources for the completion of its constituent activities, which are interrelated and which must be accomplished to achieve the objectives of the project. Project management is evolved to coordinate and control all project activities in an efficient and cost effective manner. The salient features of a project are:

- A project has identifiable beginning and endpoints.
- Each project can be broken down into a number of identifiable activities which will consume time and other resources during their completion.
- A project is scheduled to be completed by a target date.
- A project is usually large and complex and has many interrelated activities.
- The execution of the project activities is always subjected to some uncertainties and risks.

Network Techniques:

Basically, PERT (Programme Evaluation Review Technique) and CPM (Critical Path Method) are project management techniques, which have been created out of the need of Western industrial and military establishments to plan, schedule and control complex projects. CPM/PERT or Network Analysis as the technique is sometimes called, developed along two parallel streams, one industrial and the other military. CPM was the discovery of M. R. Walker of E. I. Du Pont de Nemours & Co. and J. E. Kelly of Remington Rand, circa 1957. The computation was designed for the UNIVAC-I computer. The first test was made in 1958, when CPM was applied to the construction of a new chemical plant. In March 1959, the method was applied to maintenance shut-down at the Du Pont works in Louisville, Kentucky. Unproductive time was reduced from 125 to 93 hours. PERT was devised in 1958 for the POLARIS missile program by the Program Evaluation Branch of the Special Projects office of the US Navy, helped by the Lockheed Missile Systems division and the Consultant firm of Booz-Allen & Hamilton. The calculations were so arranged so that they could be carried out on the IBM Naval Ordinance Research Computer (NORC) at Dahlgren, Virginia.

The network techniques of project management have developed in an evolutionary way in many years. Up to the end of 18^{th} century, the decision making in general and project management in particular was intuitive and depended primarily on managerial capabilities, experience, judgment and academic background of the managers. It was only in the early of 1900's that the pioneers of scientific management started developing the scientific management techniques. The forerunner to network techniques, the Gantt chart was developed, during World War I, by Henry L Gantt, for the purpose of productionscheduling.

NETWORK CONSTRUCTION

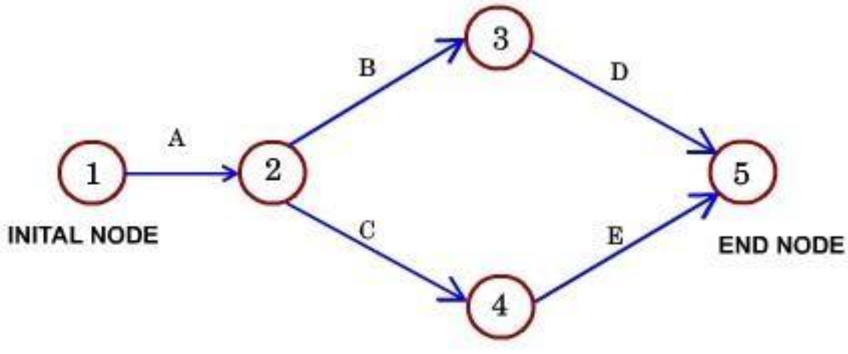
A network is the graphical representation of the project activities arranged in a logical sequence and depicting all the interrelationships among them. A network consists of activities and events.
The activities can be further classified into three categories:

1. **Predecessor activity:** An activity which must be completed before one or more other activities start is known as predecessor activity.
2. **Successor activity:** An activity which started immediately after one or more of the activities are completed is known as a successor activity.
3. **Dummy activity:** An activity which does not consume either any resource or time is known as a dummy activity. A dummy activity is depicted by dotted line in the network diagram.

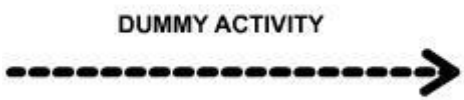
Activity
An activity is a physically identifiable part of a project, which consumes both time and resources. Activity is represented by an arrow in a network diagram.

The head of an arrow represents the start of activity and the tail of arrow represents its end. Activity description and its estimated completion time are written along the arrow.

An activity in the network can be represented by a number of ways:
(i) By numbers of its head and tail events (i.e. 10-20 etc.),and
(ii) By a letter code (i.e. A, B etc.). All those activities, which must be completed before the start of activity under consideration, are called its predecessor activities. All those activities, which have to follow the activity under consideration, are called its successor activities.



An activity, which is used to maintain the pre-defined precedence relationship only during the



construction of the project network, is called a dummy activity. Dummy activity is represented by a dotted arrow and does not consume any time and resource.

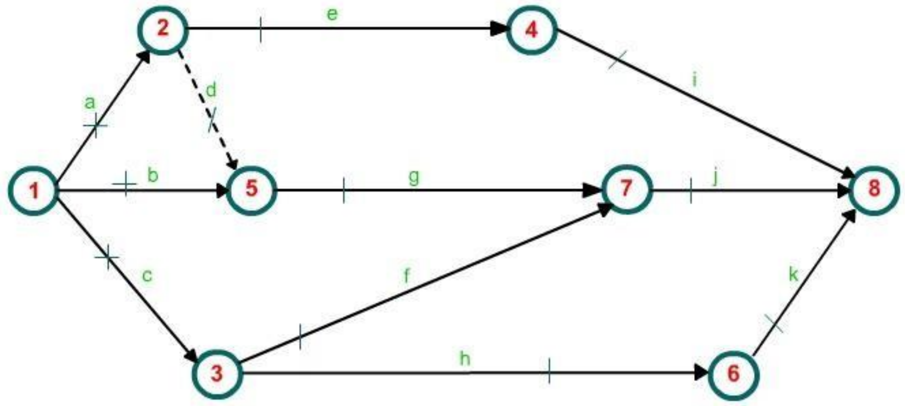
An unbroken chain of activities between any two events is called a path.

Event

An event represents the accomplishment of some task. In a network diagram, beginning and ending of an activity are represented as events. Each event is represented as a node in a network diagram. An event does not consume any time or resource. Each network diagram starts with an initial event and ends at a terminal event. Each node is represented by a circle and numbered by using the Fulkerson's Rule. Following steps are involved in the numbering of the nodes:

- The initial event, which has all outgoing arrows and no incoming arrow, is numbered as1.
- Delete all the arrows coming out from the node just numbered (i.e. 1). This step will create some more nodes (at least one) into initial events. Number these events in ascending order (i.e. 2, 3 - etc.).
- Continue the process until the final or terminal node which has all arrows coming in, with no arrow going out, is numbered.

An illustration of Fulkerson's Rule of numbering the events is shown below.

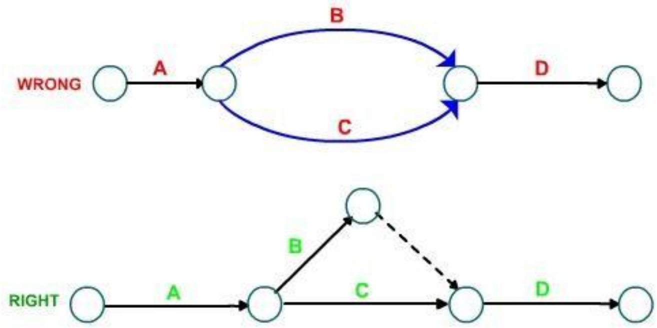


As a recommendation it must be noted that most of the projects are liable for modifications, and hence there should be a scope of adding more events and numbering them without causing any inconsistency in the network. This is achieved by skipping the numbers (i.e. 10, 20, 30).

Rules for drawing network diagram

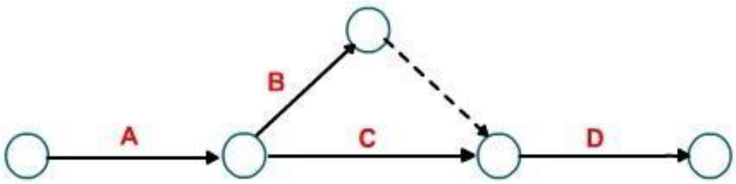
Rule1 Each activity is represented by one and only one arrow in the network.

Rule2 No two activities can be identified by the same end events.

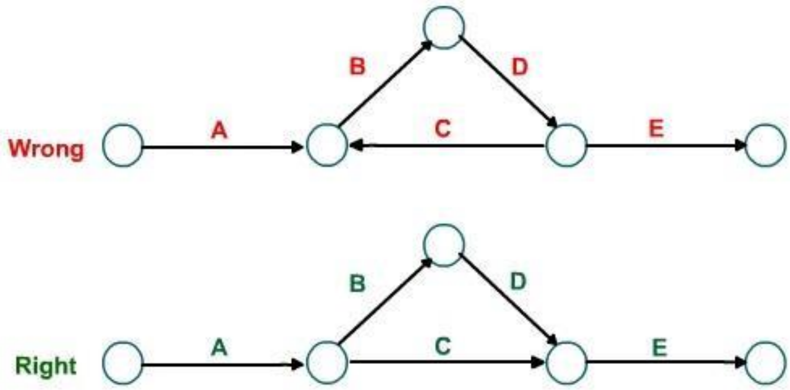


Rule3 Precedence relationships among all activities must always bemaintained.

Rule4 Dummy activities can be used to maintain precedence relationships onlywhen actually required. Their use should be minimized in the networkdiagram.



Rule5 Looping among the activities must beavoided.



Five useful questions to ask when preparing an activity network are:

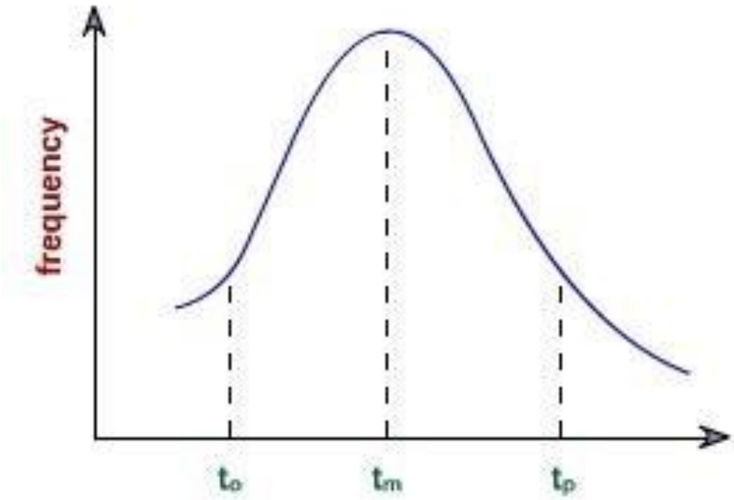
- Is this a Start Activity?
- Is this a Finish Activity?
- What Activity Precedes this?
- What Activity Follows this?
- What Activity is Concurrent with this?

PERT and CPM have been used for a variety of projects, including the following types.

1. Construction of a new plant
2. Research and development of a new product
3. Space exploration projects
4. Movie productions
5. Building a ship
6. Government-sponsored projects for developing a new weapons system
7. Relocation of a major facility
8. Maintenance of a nuclear reactor
9. Installation of a management information system
10. Conducting an advertising campaign

The CPM (critical path method) system of networking is used, when the activity time estimates are deterministic in nature. For each activity, a single value of time, required for its execution, is estimated. Time estimates can easily be converted into cost data in this technique. CPM is an activity oriented technique.

The PERT (Project Evaluation and Review Technique) technique is used, when activity time estimates are stochastic in nature. For each activity, three values of time (optimistic, most likely, pessimistic) are estimated. Optimistic time (t_o) estimate is the shortest possible time required for the completion of activity. Most likely time (t_m) estimate is the time required for the completion of activity under normal circumstances. Pessimistic time (t_p) estimate is the longest possible time required for the completion of activity. In PERT β-distribution is used to represent these three time estimates.



As PERT activities are full of uncertainties, times estimates cannot easily be converted in to cost data. PERT is an event oriented technique.

COMPARISON CHART forPERT/CPM

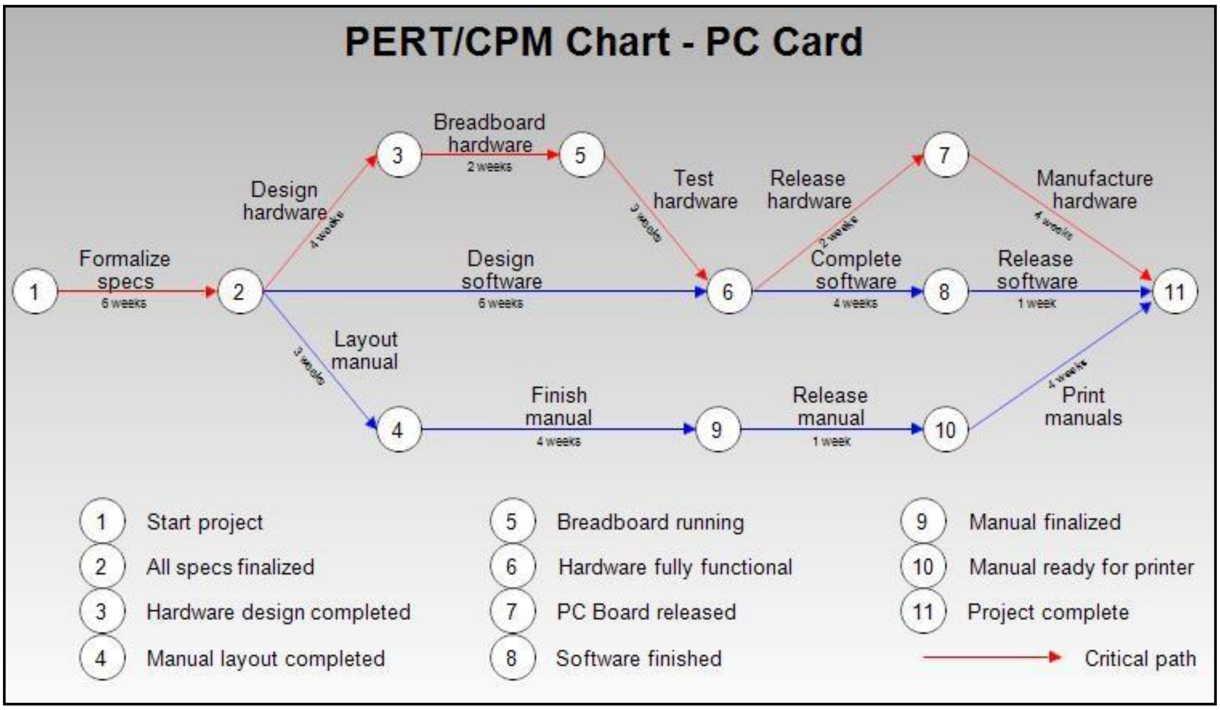
Basis for Comparison	PERT	CPM
Meaning	It is a project management technique, used to manage uncertain activities of a project.	It is a statistical technique of project management that manages well defined activities of a project.
What is it?	A technique of planning and control of time.	A method to control cost and time.
Orientation	Event-oriented	Activity-oriented
Evolution	Evolved as Research & Development project	Evolved as Construction project
Model	Probabilistic Model	Deterministic Model
Focuses on	Time	Time-cost trade-off
Estimates	Three time estimates	One time estimate
Appropriate for	High precision time estimate	Reasonable time estimate
Management of	Unpredictable Activities	Predictable activities
Nature of jobs	Non-repetitive nature	Repetitive nature
Critical and Non-critical activities	No differentiation	Differentiated
Suitable for	Research and Development Project	Non-research projects like civil construction, ship building etc.

Crashing concept	Not Applicable	Applicable

Key Differences Between PERT and CPM

The most important differences between PERT and CPM is provided below:

1. PERT is a project management technique, whereby planning, scheduling, organising, coordinating and controlling of uncertain activities is done. CPM is a statistical technique of project management in which planning, scheduling, organising, coordination and control of well-defined activities takes place.
2. PERT is a technique of planning and control of time. Unlike CPM, this is a method to control costs and time.
3. While PERT is evolved as research and development project, CPM evolved as construction project.
4. PERT is set according to events while CPM is aligned towards activities.
5. A deterministic model is used in CPM. Conversely, PERT uses probabilistic model.
6. There are three times estimates in PERT i.e. optimistic time (t_o), most likely time t_m, pessimistic time (t_p). On the other hand, there is only one estimate in CPM.
7. PERT technique is best suited for a high precision time estimate, whereas CPM is appropriate for a reasonable time estimate.
8. PERT deals with unpredictable activities, but CPM deals with predictable activities.
9. PERT is used where the nature of the job is non-repetitive. In contrast to, CPM involves the job of repetitive nature.
10. There is a demarcation between critical and non-critical activities in CPM, which is not in the case of PERT.
11. PERT is best for research and development projects, but CPM is for non-research projects like construction projects.
12. Crashing is a compression technique applied to CPM, to shorten the project duration, along with least additional cost. The crashing concept is not applicable to PERT.



THE CRITICAL PATH and ACTIVITY SLACK

- A path through a network is one of the routes following the arrows (arcs) from the start node to the finish node.
- The length of a path is the sum of the (estimated) durations of the activities on thepath.
- The (estimated) project duration or project completion time equals the length of the longest path through the project network.
- This longest path is called the critical path. (If more than one path ties for the longest, they are all critical paths.)
- Activity Slack is the amount of time that noncritical activities can be delayed without increasing the project completion time.
- The critical path will always consist of activities with zero slack.

Forward Pass (Earliest Occurrence Times)
The starting and finishing times of each activity if no delays occur anywhere in the project are called the earliest start time and the earliest finish time of the activity.
These times are represented by the symbols

ES = earliest start time for a particular activity
EF = earliest finish time for a particular activity
Where,
EF = ES + (estimated) duration of the activity.

Earliest Start Time Rule

The earliest start time of an activity is equal to the *largest* of the earliest finish times of its immediate predecessors. In symbols,

ES = largest EF of the immediate predecessors

Backward Pass (Latest Occurrence Times)
The **latest start time** for an activity is the latest possible time that it can start without delaying the completion of the project (so the FINISH node still is reached at its earliest finish time), assuming no subsequent delays in the project. The latest finish time has the corresponding definition with respect to finishing the activity.
In symbols,

LS = latest start time for a particular activity,
LF = latest finish time for a particular activity,
Where,
LS = LF - (estimated) duration of the activity.

Latest Finish Time Rule

The latest finish time of an activity is equal to the *smallest* of the latest start times of its immediate successors. In symbols,

LF = smallest LS of the immediate successors

The PERT Three-Estimate Approach
The three estimates to be obtained for each activity are

Most likely estimate (m)	=	estimate of the most likely value of the duration
Optimistic estimate (o)	=	estimate of the duration under the most favorable conditions
Pessimistic estimate (p)	=	estimate of the duration under the most un favorable conditions

In PERT expected time of an activity is determined by using the below given formula:

$$t_e = \frac{(t_o + 4t_m + t_p)}{6}$$

The variance of an activity can be calculated as:

$$\sigma^2 = \left[\frac{(t_p - t_o)}{6}\right]^2$$

Determination of Floats

Floats are the slack times available within the allotted span of the non-critical activity. The most common are the free float and the free float.

a. **Total Float:** It is the activity represents the amount of time by which an activity can be delayed without delay in the project completion date. It is the positive difference between the earliest finish time and the latest finish time. or the positive difference between the earliest start time and the latest start time of an activity depending upon which is defined.

Total Float = [LS - ES] (or) [LF - EF] (or) [LF - ES - Duration of the Activity]

b. **Free Float:** It is the portion of total float within which an activity can be manipulated without affecting the float of subsequent activities. It is computed for an activity by subtracting the head event slack from its total float.

Free Float = [ES of succeeding activity - EF of this activity]

c. **Independent Float:** It is that portion of total float within which an activity can be delayed for start without affecting floats of the preceding activities. It is computed by subtracting the tail event slack from the free float of the activity. If the result is negative, it is taken as zero, which causes a reduction in the float of the successor activities.

Independent Float = [ES of succeeding activity - LF of preceding activity] - Duration of this activity

d. **Interfering Float:** Interfering float can be defined as that part of the total float It is the difference between the latest finish time of the activity under consideration and the earliest start time of the following activity, or zero, whichever is larger.

Interfering Float = [LF - ES of Successor] (or) [Zero] whichever is higher

APPROXIMATING THE PROBABILITY OF MEETING THE DEADLINE

Calculation of $\mu_p$and : $_p$

Because of simplifying approximation 1, the *mean* of the probability distribution of project duration is approximately

μ_p= sum of the *means* of the durations for the activities on the mean critical path.

Because of both simplifying approximations 1 and 2, the *variance* of the probability distribution of project duration is approximately

σ_p^2 = sum of the variances of the durations for the activities on the mean critical path.

T = project duration (in weeks), which has (approximately) a normal distribution with mean $\mu_p$and variance σ^2_p

d = deadline for the project

$$Z = \frac{d - \mu_p}{S_t}$$

PROJECT CRASHING

Crashing is a schedule compression technique used to reduce or shorten the project schedule. The Project Management can various measures to accomplish this goal. Some of the common methods used are

- **Adding additional resources to the critical path tasks:** This option has various constraints such as the securing of the budget to add the resources, and the availability of the resources.
- **Reduce the project requirements or scope:** This can be done only if the sponsor and major stakeholders agree to reduce the scope

Definitions:

1. Crashing is the technique to use when fast tracking has not saved enough time on the schedule. It is a technique in which resources are added to the project for the least cost possible. Cost and schedule tradeoffs are analyzed to determine how to obtain the greatest amount of compression for the least incremental cost.
2. Crashing refers to a particular variety of project schedule compression which is performed for the purposes of decreasing total period of time (also known as the total project schedule duration). The diminishing of the project duration typically take place after a careful and thorough analysis of all possible project duration minimization alternatives in which any and all methods to attain the maximum schedule duration for the least additional cost.
3. When we say that an activity will take a certain number of days or weeks, what we really mean is this activity normally takes this many Project Management Triangle days or weeks. We could make it take less time, but to do so would cost more money. Spending more money to get something done more quickly is called–crashing‖.There are various methods of project schedule crashing, and the decision to crash should only take place after you‘ve carefully analyzed all of the possible alternatives. The key is to attain maximum decrease in schedule time with minimum cost.
4. Crashing the schedule means to throw additional resources to the critical path without necessarily getting the highest level of efficiency.
5. Crashing is another schedule compression technique where you add extra resources to the project to compress the schedule. In crashing, you review the critical path and see which activities can be completed by adding extra resources. You try to find the activities that can be reduced the most by adding the least amount of cost. Once you find those activities, you will apply the crashing technique.

Time-Cost Trade-Offs For Individual Activities (Crashing)

The first key concept for this approach is that of *crashing.*

Crashing an activity refers to taking special costly measures to reduce the duration of an activity below its normal value. These special measures might include using overtime, hiring additional temporary help, using special time-saving materials, obtaining special equipment, etc. **Crashing the project** refers to crashing a number of activities in order to reduce the duration of the project below its normal value.

The **CPM method of time-cost trade-offs** is concerned with determining how much (if any) to crash each of the activities in order to reduce the anticipated duration of the project to a desired value.

The data necessary for determining how much to crash particular activities are given by the *time-cost graph* for the activity. Figure 10.11 shows a typical time-cost graph. Note the two key points on this graph labelled *Normal* and *Crash.*

The **normal point** on the time-cost graph for an activity shows the time (duration) and cost of the activity when it is performed in the normal way. The **crash point** shows the time and cost when the activity is *fully crashed*, i.e., it is fully expedited with no cost spared to reduce its duration as much as possible. As an approximation, CPM assumes that these times and costs can be reliably predicted without significant uncertainty.

Normal Time(NT)	The expected time to complete an activity
Normal Cost(NC)	The cost to complete the activity in its normal time
Crash Time(CT)	The shortest possible time in which the activity can be completed
Crash Cost(CC)	The cost to complete the activity in the shortest possible time (i.e. the cost to complete the activity in its crash time).

$$\textbf{Crash Cost per Time Period (slope)} = \frac{\mathbf{CC-NC}}{\mathbf{NT-CT}}$$

Steps to Approach the Crashing Network

1. The crashing activity starting at least slope value in activity to be selected in critical path (CPM).
2. The project duration is crashing are starting in critical path because of the project duration is depend in critical path to be selected or work.
3. The crashing activity are starting the number of days / time is reduced in selected path (CPM) is directly dependent of unselected path. Hence the unselected path is not away or overhead of the critical path.
4. The unselected path is less than or equal to the selected path at crashing duration.

Time-cost trade-offs for crashing activities

Queuing Theory: Queuing theory is a branch of mathematics that studies and models the act of waiting in lines. This paper will take a brief look into the formulation of queuing theory along with examples of the models and applications of their use. The goal of the paper is to provide the reader with enough background in order to prop- erly model a basic queuing system into one of the categories we will look at, when possible. Also, the reader should begin to understand the basic ideas of how to de- termine useful information such as average waiting times from a particular queuing system.

Queuing Structure and basic component of an Queuing Model:

Distributions in Queuing Model:

To begin understanding queues, we must first have some knowledge of probability theory. In particular, we will review the exponential and Poisson probability distributions.

Exponential and Poisson Probability Distributions. The exponential distribution with parameter λ is given by $\lambda e^{-\lambda t}$ for $t \geq 0$. If T is a random variable that represents interarrival times with the exponential distribution, then $P(T \leq t) = 1 - e^{-\lambda t}$ and $P(T > t) = e^{-\lambda t}$.

This distribution lends itself well to modeling customer inter arrival times or service times for a number of reasons. The first is the fact that the exponential function is a strictly decreasing function of *t*. This means that after an arrival has occurred, the amount of waiting time until the next arrival is more likely to be small than large. Another important property of the exponential distribution is what is known as the no-memory property. The no-memory property suggests that the time until the next arrival will never depend on how much time has already passed. This makes intuitive sense for a model where we're measuring customer arrivals because the customers' actions are clearly independent of one another.

It's also useful to note the exponential distribution's relation to the Poisson distribution. The Poisson distribution is used to determine the probability of a

certain number of arrivals occurring in a given time period. The Poisson distribution with parameter λ is given by

$$\frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

where *n* is the number of arrivals. We find that if we set $n = 0$, the Poisson distribution gives us $e^{-\lambda t}$

which is equal to $P(T > t)$ from the exponential distribution.

The relation here also makes sense. After all, we should be able to relate the probability that zero arrivals will occur in a given period of time with the probability that an inter arrival time will be of a certain length. The inter arrival time here, of course, is the time between customer arrivals, and thus is a period of time with zero arrivals.

With these distributions in mind, we can begin defining the input and output processes of a basic queuing system, from which we can start developing the model further.

Different in Queuing Model with FCFS:

It is easy for one to think of all queues operating like a grocery checkout line. That is to say, when an arrival occurs, it is added to the end of the queue and service is not performed on it until all of the arrivals that came before it are served in the order they arrived. Although this a very common method for queues to be handled, it is far from the only way. The method in which arrivals in a queue get processed is known as the queuing discipline. This particular example outlines a first-come-first-serve discipline, or an FCFS discipline. Other possible disciplines include last-come-first-served or LCFS, and service in random order, or SIRO. While the particular discipline chosen will likely greatly affect waiting times for particular customers (nobody wants to arrive early at an LCFS discipline), the discipline generally doesn't affect important outcomes of the queue itself, since arrivals are constantly receiving service regardless.

Queue Discipline, Single and Multiple service station with finite and infinite population:

In many queues, it is useful to determine various waiting times and queue sizes for Particular components of the system in order to make judgments about how the system should be run. Let us define L to be the average number of customers in the queue at any given moment of time assuming that the steady-state has been reached. We can break that down into L_q, the average number of customers waiting in the queue, and L_s, the average number of customers in service. Since customers in the system can only either be in the queue or in service, it goes to show that

$L = L_q + L_s$.

Likewise, we can define W as the average time a customer spends in the queuing system. W_q is the average amount of time spent in the queue itself and W_s is the average amount of time spent in service. As was the similar case before, $W = W_q + W_s$. It should be noted that all of the averages in the above definitions are the steady-state averages. Defining λ as the arrival rate into the system, that is, the number of customers arriving the system per unit of time, it can be shown that

$$L = \lambda W$$

$$L_q = \lambda W_q \; L_s = \lambda W_s$$

GAME THEORY

Game theory was developed for decision making under conflicting situations where there are one or more opponents (players). The games like chess, poker, etc. have the characteristics of competition and are played according to some definite rules. Game theory provides optimal solutions to such games, assuming that each of the players wants to maximize his profit or minimize his loss. Game theory has applications in a variety of areas including business and economics. In 1994, the Nobel Prize for Economic Sciences was won by John F. Nash, Jr., John C. Harsanyi, and Rein hard Selton for their analysis of equilibrium in the theory of non-cooperative games. Later, in 2005, Robert J. Aumann and Thomas C. Schelling won the Nobel Prize for Economic Sciences for enhancing our understanding of conflict and cooperation through game theory analysis.

Definition

–Game theory is a mathematical technique helpful in making decisions in situations of conflicts, where the success of one part depends at the expense of others, and where the individual decision maker is not in complete control of the factors influencing the outcome.

- Robert Mockler

–Game theory, more properly the theory of games of strategy, is a mathematical method of analyzing a conflict. The alternative is not between this decision or that decision, but between this strategy or that strategy to be used against the conflicting interest‖.

- William G. Nelson

Assumptions for a Competitive Game

Game theory helps in finding out the best course of action for a firm in view of the anticipated counter moves from the competing organizations. A competitive situation is a competitive game if the following properties hold:

1. The number of competitors is finite, say.
2. A finite set of possible courses of action is available to each of the competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result, no competitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

Managerial Applications of the Theory of Games

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1. Analysis of the market strategies of a business organization in the long run.
2. Evaluation of the responses of the consumers to a new product.
3. Resolving the conflict between two groups in a business organization.
4. Decision making on the techniques to increase market share.
5. Material procurement process.
6. Decision making for transportation problem.
7. Evaluation of the distribution system.
8. Evaluation of the location of the facilities.
9. Examination of new business ventures and
10. Competitive economic environment.

KEY CONCEPTS IN THE THEORY OFGAMES

Players	The competitors or decision makers in a game are called the players of the game.
Strategies	The alternative courses of action available to a player are referred to as his strategies.
Pay off	The outcome of playing a game is called the payoff to the concerned player.
Optimal Strategy	A strategy by which a player can achieve the best pay off is called the optimal strategy for him.
Zero-sum game	A game in which the total payoff to all the players at the end of the game is zero is referred to as a zero-sum game.
Non-zero sum game	Games with–less than complete conflict of interest‖are called non-zero sum games. The problems faced by a large number of business organizations come under this category. In such games, the gain of one player in terms of his success need not be completely at the expense of the otherplayer.
Payoff matrix	The tabular display of the payoffs to players under various alternatives is called the payoff matrix of the game.
Pure strategy	If the game is such that each player can identify one and only one strategy as the optimal strategy in each play of the game, then that strategy is referred to as the best strategy for that player and the game is referred to as a game of pure strategy or a pure game.

Mixed strategy	If there is no one specific strategy as the ‗best strategy' for any player in a game, then the game is referred to as a game of mixed strategy or a mixed game. In such a game, each player has to choose different alternative courses of action from time to time.
N-person game	A game in which N-players take part is called an N-person game.
Maximin-Minimax Principle	The maximum of the minimum gains is called the maximin value of the game and the corresponding strategy is called the maximin strategy. Similarly the minimum of the maximum losses is called the minimax value of the game and the corresponding strategy is called the minimax strategy. If both the values are equal, then that would guarantee the best of the worst results.
Negotiable or cooperative game	If the game is such that the players are taken to cooperate on any or every action which may increase the payoff of either player, then we call it a negotiable or cooperative game.
Non-negotiable or non-cooperative game	If the players are not permitted for coalition then we refer to the game as a non-negotiable or non-cooperative game.
Saddle point	A saddle point of a game is that place in the payoff matrix where the maximum of the row minima is equal to the minimum of the column maxima. The payoff at the saddle point is called the value of the game and the corresponding strategies are called the pure strategies.
Dominance	One of the strategies of either player may be inferior to at least one of the remaining ones. The superior strategies are saidto dominate the inferior ones.

TYPES OF GAMES

There are several classifications of a game. The classification may be based on various factors such as the number of participants, the gain or loss to each participant, the number of strategies available to each participant, etc. Some of the important types of games are enumerated below.

Two person games andn-persongames

In two person games, there are exactly two players and each competitor will have a finite number of strategies. If the number of players in a game exceeds two, then we refer to the gameas n-persongame.

Zero sum game and non-zero sumgame

If the sum of the payments to all the players in a game is zero for every possible outcome of the game, then we refer to the game as a zero sum game. If the sum of the payoffs from any play of the game is either positive or negative but not zero, then the game is called a non-zero sum game.

Games of perfect information and games of imperfect information

It is the one in which each player can find out the strategy that would be followed by his opponent. On the other hand, a game of imperfect information is the one in which no player can know in advance what strategy would be adopted by the competitor and a player has to proceed in his game with his guess worksonly.

Games with finite number of moves / players and games with unlimitednumber of moves

A game with a finite number of moves is the one in which the number of moves for each player is limited before the start of the play. On the other hand, if the game can be continued over an extended period of time and thenumber of moves for any player has no restriction, then we call it a game with unlimited number ofmoves.

Constant-sum games

If the sum of the game is not zero but the sumof the payoffs to both players in each case is

2x2 two person game and 2xn and mx2 games

constant, then we call it a constant sum game. It is possible to reduce such a game to a zero-sum game.

When the number of players in a game is two and each player has exactly two strategies, the game is referred to as 2x2 two person game. A game in which the first player has precisely two strategies and the second player has three or more strategies is called an 2xn game. A game in which the first player has three or more strategies and the second player has exactly two strategies is called an mx2game.

3x3 andlargegames

When the number of players in a game is two and each player has exactly three strategies, we call it a 3x3 two person game. Two-person zero sum games are said to be larger if each of the two players has 3 or more choices. The examination of 3x3 and larger games is involves difficulties. For such games, the technique of linear programming can be used as a method of solution to identify the optimum strategies for the two players.

Non-constantgames

Consider a game with two players. If the sum of the payoffs to the two players is not constant in all the plays of the game, then we call it a non-constant game. Such games are divided into negotiable or cooperative games and non-negotiable or non-cooperative games.

Value of the Game:

A game with only two players, say player A and player B, is called a two-person zero sum game if the gain of the player A is equal to the loss of the player B, so that the total sum is zero.

Payoff matrix

When players select their particular strategies, the payoffs (gains or losses) can be represented in the form of a payoff matrix. Since the game is zero sum, the gain of one player is equal to the loss of other and vice-versa. Suppose A has m strategies and B has n strategies.

Assumptions for two-person zero sum game

For building any model, certain reasonable assumptions are quite necessary. Some assumptions for building a model of two-person zero sum game are listed below.

a) Each player has available to him a finite number of possible courses of action. Sometimes the set of courses of action may be the same for each player. Or, certain courses of action may be available to both players while each player may have certain specific courses of action which are not available to the other player.
b) Player A attempts to maximize gains to himself. Player B tries to minimize losses to himself.
c) The decisions of both players are made individually prior to the play with no communication between them.
d) The decisions are made and announced simultaneously so that neither player has an advantage resulting from direct knowledge of the other player‘s decision.
e) Both players know the possible payoffs of themselves and their opponents.

Minimax and Maximin Principles

The selection of an optimal strategy by each player without the knowledge of the competitor‘s strategy is the basic problem of playing games. The objective of game theory is to know how these players must select their respective strategies, so that they may optimize their payoffs. Such a criterion of decision making is referred to as minimax-maximin principle. This principle in games of pure strategies leads to the best possible selection of a strategy for both players.

Maximin Principle: For player A the minimum value in each row represents he least gain (payoff) to him, if he chooses his particular strategy. These are written in the matrix by row minima. He will then select the strategy that gives the largest gain among the row minimum values. This choice of player A is called the maximin principle, and the corresponding gain is called the maximin value of the game.

Minimax Principle: For player B, who is assumed to be the looser, the maximum value in each column represents the maximum loss to him, if he chooses his particular strategy. These are written in the payoff matrix by column maxima. He will then select the strategy that gives the minimum loss among the column maximum values. This choice of player A is called the minimax principle, and the corresponding loss is called the minimax value of the game.

Optimal Strategy: A course of action that puts any player in the most preferred position, irrespective of the course of action his competitor(s) adopt, is called as optimal strategy. In other words, if the maximin value equals the minimax value, then the game is said to have a saddle (equilibrium) point and the corresponding strategies are called optimal strategies.

Value of the Game: This is the expected payoff at the end of the game, when each player uses his optimal strategy, i.e. the amount of payoff, V, at an equilibrium point. A game may have more than one saddle points. A game with no saddle point is solved by choosing strategies with fixed probabilities.

Games with Mixed Strategies

In certain cases, no pure strategy solutions exist for the game. In other words, saddle point does not exist. In all such game, both players may adopt an optimal blend of the strategies called Mixed Strategy to find a saddle point. The optimal mix for each player may be determined by assigning each strategy a probability of it being chosen. Thus these mixed strategies are probabilistic combinations of available better strategies and these games hence called Probabilistic games.

The probabilistic mixed strategy games without saddle points are commonly solved by any of the following methods

Method	Applicable to
Algebraic, matrix, and Analytical Method	2x2 games
Graphical Method	2x2, mx2 and 2xn games
Simplex Method	2x2, mx2, 2xn and mxn games

Graphical method

The graphical method is used to solve the games whose payoff matrix has

- Two rows and n columns (2 xn)
- m rows and two columns (m x2)

Algorithm for solving 2 x n matrix games

- Draw two vertical axes 1 unit apart. The two lines are $X_1 = 0$, $X_1 = 1$
- Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second row in the payoff matrix on the vertical line $x_1 = 0$.
- The point a_{1j} on axis $X_1 = 1$ is then joined to the point a_{2j} on the axis $X_1 = 0$ to give a straight line. Draw ‗n' straight lines for j=1, 2… n and determine the highest point of the lower envelope obtained. This will be the maximin point.
- The two or more lines passing through the maximin point determines the required 2x2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Algorithm for solving m x 2 matrix games

- Draw two vertical axes 1 unit apart. The two lines are $X_1 = 0$, $X_1 = 1$
- Take the points of the first row in the payoff matrix on the vertical line $X_1 = 1$ and the points of the second row in the payoff matrix on the vertical line $X_1 = 0$.
- The point a_{1j} on axis $X_1 = 1$ is then joined to the point a_{2j} on the axis $X_1 = 0$ to give a straight line. Draw ‗n' straight lines for j=1, 2… n and determine the lowest point of the upper envelope obtained. This will be the minimax point.
- The two or more lines passing through the minimax point determines the required 2x2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Suddle point:

In a game, sometimes a strategy available to a player might be found to be preferable to some other strategy / strategies. Such a strategy is said to dominate the other one(s). The rules of dominance are used to reduce the size of the payoff matrix. These rules help in deleting certain rows and/or columns of the payoff matrix, which are of lower priority to at least one of the remaining rows, and/or columns in terms of payoffs to both the players. Rows / columns once deleted will never be used for determining the optimal strategy for both the players. This concept of domination is very usefully employed in simplifying the two – person zero sum games without saddle point. In general the following rules are used to reduce the size of payoff matrix.

1. For deleting the ineffective rows and columns, the following general rules are to be followed: 1. If all the elements of a row (say, i^{th} row) of a payoff matrix are less than or equal to the corresponding elements of any other row (say, j^{th} row) then the player A will never choose the i^{th} strategy. Therefore, i^{th} row is dominated by j^{th} row. Hence delete ith row.
2. If all the elements of a column (say, j^{th} column) are greater than or equal to the corresponding elements of any other column (say, i^{th} column) then j^{th} column is dominated by i^{th} column. So

delete j^{th} column.

3. A pure strategy of a player may also be dominated if it is inferior to some convex combination of two or more pure strategies. As a particular case, if all the elements of a column are greater than or equal to the average of two or more other columns then this column is dominated by the group of columns. Similarly, if all the elements of row are less than or equal to the average of two or more rows then this row is dominated by the group of rows.